Atomic Theory and the Description of Nature

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Cambridge

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I

FOUR ESSAYS

With an Introductory Survey

BY

NIELS BOHR

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Of the four articles contained in this volume the first two appeared originally in English in *Nature* in 1925 and 1927, while the third appeared in German in *Die Naturwissenschaften* in 1929 and the fourth in Danish in *Fysisk Tidsskrift* in 1929. The Introductory Survey originally appeared in Danish in the Year Book of Copenhagen University for 1929 together with a Danish translation of the first three articles, the Addendum being first included in the German edition of all four articles published by Jul. Springer, Berlin, in 1931. I am indebted to Prof. Rud Nielsen and Dr Urquhart for their valuable help in preparing the present English translation, and to the Syndics of the Cambridge University Press for their kind interest in this edition, as well as for their courtesy in arranging for this volume to be followed by another containing a number of later essays on the same subject, in which the general point of view is further developed.

N. BOHR

COPENHAGEN
February 1934

PREFACE TO THE 1961 REISSUE

I am indebted to the Cambridge University Press for the suggestion to re-issue this collection of essays, which for some time has been out of print.

The articles were written at a time when the programme of developing a comprehensive treatment of atomic problems, on the basis of Planck's original discovery of the universal quantum of action, had obtained a solid foundation by the establishment of a proper mathematical formalism.

As is well known, the discussion of the epistemological aspects of quantum physics continued during the following years, and general consent has still not been reached. In the course of this discussion the attitude advocated in the articles was further developed, and especially a more adequate terminology was introduced to express the radical departure from ordinary pictorial description and accustomed demands of physical explanation. A collection of essays from this phase of the discussion has recently been published under the title *Atomic Physics and Human Knowledge* (John Wiley and Sons, New York, 1958).

Even if the old articles, which are here reprinted, thus contain utterances which now may be formulated in a more precise manner, acquaintance with the early discussions might, however, be helpful for a full appreciation of the new situation in natural philosophy with which the modern development of physics has confronted us.

NIELS BOHR

COPENHAGEN
January 1961

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Introductory Survey

(1929)

The task of science is both to extend the range of our experience and to reduce it to order, and this task presents various aspects, inseparably connected with each other. Only by experience itself do we come to recognize those laws which grant us a comprehensive view of the diversity of phenomena. As our knowledge becomes wider, we must always be prepared, therefore, to expect alterations in the points of view best suited for the ordering of our experience. In this connection we must remember, above all, that, as a matter of course, all new experience makes its appearance within the frame of our customary points of view and forms of perception. The relative prominence accorded to the various aspects of scientific inquiry depends upon the nature of the matter under investigation. In physics, where our problem consists in the co-ordination of our experience of the external world, the question of the nature of our forms of perception will generally be less acute than it is in psychology where it is our own mental activity which is the object under investigation. Yet occasionally just this "objectivity" of physical observations becomes particularly suited to emphasize the subjective character of all experience. There are many examples of this in the history of science. I need only mention the great significance that the investigation of acoustical and optical phenomena, the physical media of our senses, has continually had in the development of psychological analysis. As another example, we may notice the rôle which the elucidation of the laws of mechanics has played in the development of the general theory of knowledge. In the latest developments of physics, this fundamental feature of science has been particularly prominent. The great extension of our experience in recent years has brought to light the insufficiency of our simple mechanical conceptions and, as a consequence, has shaken the foundation on which the customary interpretation of observations was based, thus throwing new light on old philosophical problems. This is true not only of the revision of the foundations of the space-time mode of description brought about by the theory of relativity, but also of the renewed discussion of the principle of causality which has emerged from the quantum theory.

The origin of the *theory of relativity* is closely bound up with the development of electromagnetic concepts, a development which, by extending the notion of force, has brought about such a profound transformation of the ideas underlying mechanics. The recognition of the relative character of the phenomena of motion, these being dependent upon the observer, already had played an essential part in the development of classical mechanics, where it served as an effective aid in the formulation of general mechanical laws. For the time being, one succeeded in giving an apparently satisfactory treatment of the questions under discussion, both from a physical as well as from a philosophical point of view. It was, in fact, first the recognition, brought about by the electromagnetic theory, of the finite velocity of propagation of all actions of force which brought the matter to a climax. It is true that it was possible, on the basis of the electromagnetic theory, to set up a causal mode of description which retained the fundamental mechanical laws of the conservation of energy and momentum, provided one ascribed energy and momentum to the fields of force themselves. However, the conception of a universal ether, which was so useful in the development of the electromagnetic theory, appeared in this theory as an absolute frame of reference for the space-time description. The unsatisfactory character of this conception, from a philosophical point of view, was strongly emphasized by the failure of all attempts to demonstrate the motion of the earth relative to this hypothetical universal ether; and this situation was not improved by the recognition that the failure of all such attempts was in complete agreement with the electromagnetic theory. It was Einstein's elucidation of the limitation which the finite velocity of propagation of all force effects, including those of radiation, imposes upon the possibilities of observation, and, therefore, upon the application of the space-time concepts, that first led us to a more liberal attitude towards these concepts, an attitude which found its most striking expression in the recognition of the relativity of the concept of simultaneity. As we know, Einstein, adopting this attitude, succeeded in tracing significant new relationships also outside the domain to which the electromagnetic theory properly applies, and in his general theory of relativity, in which the effects of gravitation no longer occupy a special position among physical phenomena, he has approached, to a quite unexpected degree, the unity in the description of nature which is the ideal of the classical physical theories.

The *quantum theory* arose out of the development of atomic conceptions, which, during the course of the last century, had increasingly provided a fruitful field for the application of mechanics and of the electromagnetic theory. In the years near the beginning of this century, however, the application of these theories to atomic problems was destined to reveal a hitherto unnoticed limitation that found its expression in Planck's discovery of the so-called quantum of action, which imposes upon individual atomic processes an element of discontinuity quite foreign to the fundamental principles of classical physics, according to which all actions may vary in a continuous manner. The quantum of action has become increasingly indispensable in the ordering of our experimental knowledge of the properties of atoms. At the same time, however, we have been forced step by step to forego a causal description of the behaviour of individual atoms in space and time, and to reckon with a free choice on the part of nature between various possibilities to which only probability considerations can be applied. The endeavours to formulate general laws for these possibilities and probabilities by a suitably limited application of the concepts of the classical theories have led recently, after a series of phases in its development, to the creation of a rational quantum mechanics by means of which we are able to describe a very wide range of experience, and which may be regarded in every respect as a generalization of the classical physical theories. In addition, we have gradually reached a complete understanding of the intimate connection between the renunciation of causality in the quantum-mechanical description and the limitation with regard to the possibility of distinguishing between phenomena and their observation, which is conditioned by the indivisibility of the quantum of action. The recognition of this situation implies an essential change in our attitude towards the principle of causality as well as towards the concept of observation.

In spite of many points in which they differ, there is a profound inner similarity between the problems met with in the theory of relativity and those which are encountered in the quantum theory. In both cases we are concerned with the recognition of physical laws which lie outside the domain of our ordinary experience and which present difficulties to our accustomed forms of perception. We learn that these forms of perception are *idealizations*, the suitability of which for reducing our ordinary sense impressions to order depends upon the practically infinite velocity of light and upon the smallness of the quantum of action. In appraising this situation, however, we must not forget that, in spite of their limitation, we can by no means dispense with those forms of perception which colour our whole language and in terms of which all experience must ultimately be expressed. It is just this state of affairs which primarily gives to the problems in question their general philosophical interest. While the finish given to our picture of the world by the theory of relativity has already been absorbed into the general scientific consciousness, this has scarcely occurred to the same extent with those aspects of the general problem of knowledge which have been elucidated by the quantum theory.

When I was requested to write a paper for the Year Book 1929 of the University of Copenhagen, I first intended to give, in the simplest possible form, an account of the new points of view brought about by the quantum theory, starting from an analysis of the elementary concepts on which our description of nature is founded. However, my occupation with other duties did not leave me sufficient time to complete such an account, the difficulty of which arose, not least, from the continuous development of the points of view in question. Sensing this difficulty, I gave up the idea of preparing a new exposition and was led to consider using instead a translation into Danish, made for this occasion, of some articles which, during recent years, I have published in foreign journals as contributions to the discussion of the problems of the quantum theory. These articles belong to a series of lectures and papers in which, from time to time, I have attempted to give a coherent survey of the state of the atomic theory at the moment. Some previous articles of this series form in some respects a background for the three articles which are reproduced here. This is particularly true of a lecture entitled "The Structure of Atoms", which was given in Stockholm in December 1922, and which was published at the time as a Supplement of *Nature*. The articles here reproduced appear formally quite independent, however. They are intimately connected with each other, in that they all discuss the latest phase in the development of the atomic theory, a phase in which the analysis of the fundamental concepts has become so prominent. The fact that the articles follow the course of the development, and thus give an immediate impression of the gradual elucidation of the concepts, may perhaps help in some measure to make the subject more easily accessible to those readers who do not belong to the narrow circle of physicists. In the following notes on the particular circumstances under which the articles appeared, I have attempted, by the addition of some guiding remarks, to facilitate a general view of the contents and, as far as possible, to make up for such shortcomings of the exposition as might present difficulties to a wider circle of readers.

The *first article* is an elaboration of a lecture delivered at the Scandinavian Mathematical Congress at Copenhagen in August 1925. It gives in a condensed form a survey of the development of the quantum theory up to that time when a new phase was being ushered in by the paper of Heisenberg which is discussed at the close of the article. The lecture deals with the application of the mechanical concepts within the atomic theory, and shows how the ordering of a vast amount of experimental data with the help of the quantum theory had prepared the way for the new development, which is characterized by the creation of rational quantum-mechanical methods. Above all, the previous development had led to the recognition of the impossibility of carrying out a coherent causal description of atomic phenomena. A conscious resignation in this respect is already implied in the form, irrational from the point of view of the classical theories, of those postulates, mentioned in the article, upon which the author based his application of the quantum theory to the problem of atomic structure. The fact that all changes in the state of an atom are described, in agreement with the requirement of the indivisibility of the quantum of action, as *individual* processes by which the atom goes over from one so-called stationary state into another stationary state and for the occurrence of which only probability considerations can be made, must, on one hand, greatly limit the field of application of the classical theories. On the other hand, the necessity of making an extensive use, nevertheless, of the classical concepts, upon which depends ultimately the interpretation of all experience, gave rise to the formulation of the so-called correspondence principle which expresses our endeavours to utilize all the classical concepts by giving them a suitable quantum-theoretical re-interpretation. The detailed analysis of the experimental data from this point of view was, however, destined to show more and more clearly that we did not then possess sufficiently adequate expedients for carrying out a strict description based upon the correspondence principle.

Owing to the special occasion on which the lecture was delivered, special emphasis has been placed in the article upon that employment of mathematical aids which is peculiar to theoretical physics. The symbolical forms of expression of mathematics are here not merely indispensable tools for describing quantitative relationships, but they furnish at the same time an essential means for the elucidation of the general qualitative points of view. The hope expressed at the conclusion of the article that mathematical analysis would again prove capable of assisting the physicist to surmount his difficulties has in the meantime been fulfilled beyond all expectations. Not only was abstract algebra destined to play a decisive part in the formulation of Heisenberg's quantum mechanics, as mentioned in the article, but the theory of differential equations—the most important of the expedients of classical physics—was almost immediately afterwards to be extensively applied to atomic problems. The point of departure for this application was the peculiar analogy between mechanics and optics upon which already Hamilton had based his important contribution to the development of the methods of classical mechanics. The significance of this analogy for the quantum theory was first pointed out by de Broglie who, in connection with Einstein's well-known theory of light quanta, compared the motion of a particle with the propagation of wave systems. As de Broglie pointed out, this comparison made it possible to give a simple geometrical meaning to the quantization rules, mentioned in the article, for the stationary states of the atoms. By a further development of these considerations, Schrödinger succeeded in reducing the quantum-mechanical problem to the solution of a certain differential equation, the so-called Schrödinger wave equation, thus providing us with a method that has played a decisive rôle in the great development which the atomic theory has undergone in the last few years.

The *second article* is an elaboration of a paper read before an international congress of physicists which took place at Como in September 1927 at the centenary of Volta's death. By this time the above-mentioned quantum-mechanical methods had reached a high degree of perfection and had demonstrated their fruitfulness in numerous applications. Yet, a divergence of opinion had arisen with regard to the physical interpretation of the methods, and this had led to much discussion.

Especially had the great success of Schrödinger's wave mechanics revived the hopes of many physicists of being able to describe atomic phenomena along lines similar to those of classical physical theories without introducing "irrationalities" of the kind which had thus far been characteristic of the quantum theory. In opposition to this view, it is maintained in the article that the fundamental postulate of the indivisibility of the quantum of action is itself, from the classical point of view, an irrational element which inevitably requires us to forego a causal mode of description and which, because of the coupling between phenomena and their observation, forces us to adopt a new mode of description designated as *complementary* in the sense that any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomena. It is pointed out that we immediately encounter this feature when considering the questions of the nature of light and of matter. It had already been emphasized in the first article that, in our description of radiation phenomena, we are faced with a dilemma as regards the choice between the wave description of the electromagnetic theory and the corpuscular conception of the propagation of light in the theory of light quanta. With regard to matter, the confirmation which, in the meantime, de Broglie's wave ideas had received by the well-known experiments on the reflection of electrons by metal crystals placed us before a quite similar dilemma, since there can be no question of giving up the idea of the individuality of the elementary particles; for this individuality forms the secure foundation on which the whole of the recent development of the atomic theory depends.

The main purpose of the article is to show that this feature of complementarity is essential for a consistent interpretation of the quantum-theoretical methods. A very significant contribution to this discussion had been given shortly before by Heisenberg, who had pointed out the close connection between the limited applicability of mechanical concepts and the fact that any measurement which aims at tracing the motions of the elementary particles introduces an unavoidable interference with the course of the phenomena and so includes an element of uncertainty which is determined by the magnitude of the quantum of action. This indeterminacy exhibits, indeed, a peculiar complementary character which prevents the simultaneous use of space-time concepts and the laws of conservation of energy and momentum, which is characteristic of the mechanical mode of description. To understand why a causal description is impracticable, however, it is essential to remember, as shown in the article, that the magnitude of the disturbance caused by a measurement is always unknown, since the limitation in question applies to any use of mechanical concepts and, hence, applies to the agencies of observation as well as to the phenomena under investigation. This very circumstance carries with it the fact that any observation takes place at the cost of the connection between the past and the future course of phenomena. As already mentioned, *the finite magnitude of the quantum of action prevents altogether a sharp distinction being made between a phenomenon and the agency by which it is observed*, a distinction which underlies the customary concept of observation and, therefore, forms the basis of the classical ideas of motion. With this in view, it is not surprising that the physical content of the quantum-mechanical methods is restricted to a formulation of statistical regularities in the relationships between those results of measurement which characterize the various possible courses of the phenomena.

It is emphasized in the article that the symbolical garb of the methods in question closely corresponds to the fundamentally unvisualizable character of the problems concerned. We come across a particularly characteristic example of the limitation imposed upon the possibility of applying mechanical ideas when we employ the concept of stationary states which, as mentioned above, even before the development of the quantum-mechanical methods, entered as an essential element in the application of the quantum theory to problems of atomic structure. As shown in the article, any use of this concept excludes the possibility of tracing the motion of the individual particles within the atom. We are here concerned with a characteristic complementarity analogous to that which we encounter when considering the questions of the nature of light and of matter. As explained in detail the concept of stationary states may indeed be said to possess, within its field of application, just as much, or, if one prefers, just as little "reality" as the elementary particles themselves. In each case we are concerned with expedients which enable us to express in a consistent manner essential aspects of the phenomena. Besides, when we use the concept of stationary states, the necessity in the quantum theory of paying attention to the delimitation of phenomena and, as emphasized already in the first paragraph of the article, of distinguishing strictly between closed and unclosed systems, is brought before us in a very instructive manner. Hence, in the case of atoms, we come upon a particularly glaring failure of the causal mode of description when accounting for the occurrence of radiation processes. While, when following the motions of free particles, we can visualize the lack of causality by considering our lack of simultaneous knowledge of the quantities entering into the classical mechanical description, the limited applicability of classical concepts is immediately evident in our account of the behaviour of atoms, since the description of the state of a single atom contains absolutely no element referring to the occurrence of transition processes, so that in this case we can scarcely avoid speaking of a choice between various possibilities on the part of the atom.

In connection with the question of the fundamental properties of the elementary particles, it may perhaps be of interest to call attention to a peculiar complementarity recently disclosed. The fact that the experiments, which so far have been explained by ascribing a magnetic moment to electrons, have been given a natural interpretation by Dirac's theory, briefly discussed in the last paragraph of the article, is, indeed, equivalent to saying that it is not possible to detect the magnetic moment of an electron by experiments based upon a direct observation of its motion. The difference between free electrons and atoms, which we come upon here, is connected with the fact that measurements of the magnetic moment of atoms involve a renunciation, in accordance with the general conditions holding for the application of the concept of stationary states, of all attempts to trace the motion of the elementary particles.

The important task, touched upon at the close of the article, of satisfying the general demand for relativity within the frame of the quantum theory, has not as yet been carried out satisfactorily. Indeed, the above-mentioned theory of Dirac, although a great step forward in this respect, has brought to light new difficulties. The recognition of these, however, may lead to the development of new points of view with regard to the profound problems presented by the very existence of elementary particles. While the present quantum-mechanical description depends upon a re-interpretation, based on the correspondence principle, of the classical electron theory, the classical theories offer no guide whatever to the understanding of the existence of the elementary particles themselves and of their specific mass and electrical charge. We must, therefore, be prepared to find that further advance into this region will require a still more extensive renunciation of features which we are accustomed to demand of the space-time mode of description than the quantum theory attack on the atomic problem has required thus far, and we must be prepared to expect new surprises with regard to the applicability of the concepts of momentum and energy.

The extensive use of mathematical symbols which is peculiar to the methods of quantum mechanics makes it difficult to give a true impression of the beauty and logical consistency of these methods without going into mathematical details. Although in the preparation of this article I have endeavoured, as far as possible, to avoid the use of mathematical artifices, yet the purpose of the lecture, delivered before a group of physicists, to open a discussion on the present tendency in the development of the quantum theory, has made it necessary to go into details which will doubtless make it difficult for readers not somewhat acquainted beforehand with the subject. However, I wish to point out that throughout the article the main emphasis has been placed upon that purely epistemological attitude which is particularly apparent in the first section and in the concluding remarks.

In the *third article*, which is a contribution to a jubilee pamphlet published in June 1929 to celebrate the fiftieth anniversary of Planck's doctorate, I have discussed in more detail the general philosophical aspects of the quantum theory. Partly in view of the regret, so widely expressed, with regard to the renunciation of a strictly causal mode of description for atomic phenomena, the writer attempts to show that the difficulties concerning our forms of perception, which arise in the atomic theory because of the indivisibility of the quantum of action, may be considered as an instructive reminder of the general conditions underlying the creation of man's concepts. The impossibility of distinguishing in our customary way between physical phenomena and their observation places us, indeed, in a position quite similar to that which is so familiar in psychology where we are continually reminded of the *difficulty of distinguishing between subject and object*. It may perhaps appear at first sight that such an attitude towards physics would leave room for a mysticism which is contrary to the spirit of natural science. However, we can no more hope to attain to a clear understanding in physics without facing the difficulties arising in the shaping of concepts and in the use of the medium of expression than we can in other fields of human inquiry. Thus, according to the view of the author, it would be a misconception to believe that the difficulties of the atomic theory may be evaded by eventually replacing the concepts of classical physics by new conceptual forms. Indeed, as already emphasized, the recognition of the limitation of our forms of perception by no means implies that we can dispense with our customary ideas or their direct verbal expressions when reducing our sense impressions to order. No more is it likely that the fundamental concepts of the classical theories will ever become superfluous for the description of physical experience. The recognition of the indivisibility of the quantum of action, and the determination of its magnitude, not only depend on an analysis of measurements based on classical concepts, but it continues to be the application of these concepts alone that makes it possible to relate the symbolism of the quantum theory to the data of experience. At the same time, however, we must bear in mind that the possibility of an *unambiguous* use of these fundamental concepts solely depends upon the self-consistency of the classical theories from which they are derived and that, therefore, the limits imposed upon the application of these concepts are naturally determined by the extent to which we may, in our account of the phenomena, disregard the element which is foreign to classical theories and symbolized by the quantum of action.

It is just this state of affairs that is so evident in the frequently discussed dilemma with regard to the properties of light and of matter. Only in terms of the classical electromagnetic theory is it at all possible to give a tangible content to the question of the nature of light and of matter. It is true that light quanta and matter waves are invaluable expedients in the formulation of the statistical laws governing such phenomena as the photo-electric effect and the interference of electron rays. However, these phenomena belong, indeed, to a domain in which it is essential to take into account the quantum of action and where an unambiguous description is impossible. The symbolical character, in this sense, of the artifices mentioned also becomes apparent in that an exhaustive description of the electromagnetic wave fields leaves no room for light quanta and in that, in using the conception of matter waves, there is never any question of a complete description similar to that of the classical theories. Indeed, as emphasized in the second article, the absolute value of the so-called phase of the waves never comes into consideration when interpreting the experimental results. In this connection, it should also be emphasized that the term "probability amplitude" for the amplitude functions of the matter waves is part of a mode of expression which, although often convenient, can, nevertheless, make no claim to possessing general validity. As mentioned above, only with the help of classical ideas is it possible to ascribe an unambiguous meaning to the results of observation. We shall, therefore, always be concerned with applying probability considerations to the outcome of experiments which may be interpreted in terms of such conceptions. Consequently, the use made of the symbolic expedients will in each individual case depend upon the particular circumstances pertaining to the experimental arrangement. Now, what gives to the quantum-theoretical description its peculiar characteristic is just this, that in order to evade the quantum of action we must use separate experimental arrangements to obtain accurate measurements of the different quantities, the simultaneous knowledge of which would be required for a complete description based upon the classical theories, and, further, that these experimental results cannot be supplemented by repeated measurements. In fact, the indivisibility of the quantum of action demands that, when any individual result of measurement is interpreted in terms of classical conceptions, a certain amount of latitude be allowed in our account of the mutual action between the object and the means of observation. This implies that a subsequent measurement to a certain degree deprives the information given by a previous measurement of its significance for predicting the future course of the phenomena. Obviously, these facts not only set a limit to the *extent* of the information obtainable by measurements, but they also set a limit to the *meaning* which we may attribute to such information. We meet here in a new light the old truth that in our description of nature the purpose is not to disclose the real essence of the phenomena but only to track down, so far as it is possible, relations between the manifold aspects of our experience.

It is against this background that we must judge the difficulties which we come upon if we attempt to give a correct impression of the content of the quantum theory and of its relation to the classical theories. As already emphasized when discussing the second article, these questions can be fully elucidated only in terms of the mathematical symbolism which has made it possible to formulate the quantum theory as a rigorous re-interpretation, based upon the idea of correspondence, of the classical theories. In view of the reciprocal symmetry peculiar to the use of the classical concepts in this symbolism, the writer in this article has preferred the term "reciprocity" to the word "complementarity", used in the preceding article to denote the relation of mutual exclusion characteristic of the quantum theory with regard to the application of the various classical concepts and ideas. Meanwhile, as the result of further discussion, it has come to my notice that the former term may be misleading because the word "reciprocity" is frequently used in the classical theories with a quite different meaning. The term "complementarity", which is already coming into use, may perhaps be more suited also to remind us of the fact that it is the combination of features which are united in the classical mode of description but appear separated in the quantum theory that ultimately allows us to consider the latter as a natural generalization of the classical physical theories. Moreover, the purpose of such a technical term is to avoid, so far as possible, a repetition of the general argument as well as constantly to remind us of the difficulties which, as already mentioned, arise from the fact that all our ordinary verbal expressions bear the stamp of our customary forms of perception, from the point of view of which the existence of the quantum of action is an irrationality. Indeed, in consequence of this state of affairs, even words like "to be" and "to know" lose their unambiguous meaning. In this connection, an interesting example of ambiguity in our use of language is provided by the phrase used to express the failure of the causal mode of description, namely, that one speaks of a free choice on the part of nature. Indeed, properly speaking, such a phrase requires the idea of an external chooser, the existence of which, however, is denied already by the use of the word nature. We here come upon a fundamental feature in the general problem of knowledge, and we must realize that, by the very nature of the matter, we shall always have last recourse to a word picture, in which the words themselves are not further analyzed. As emphasized in the article, we must, indeed, remember that the nature of our consciousness brings about a complementary relationship, in all domains of knowledge, between the analysis of a concept and its immediate application.

The reference to certain psychological problems in the latter part of the article has a twofold purpose. The analogies with some fundamental features of the quantum theory, exhibited by the laws of psychology, may not merely make it easier for us to adjust ourselves to the new situation in physics, but it is perhaps not too ambitious to hope that the lessons we have learned from the very much simpler physical problems will also prove of value in our endeavours to obtain a comprehensive survey of the more subtle psychological questions. As stressed in the article, it is clear to the writer that for the time being we must be content with more or less appropriate analogies. Yet it may well be that behind these analogies there lies not only a kinship with regard to the epistemological aspects, but that a more profound relationship is hidden behind the fundamental biological problems which are directly connected to both sides. Although it cannot yet be said that the quantum theory has contributed essentially to the elucidation of the latter problems, still there is much which indicates that we are concerned here with questions which closely approach the circle of ideas of the quantum theory. Indeed, living organisms are first of all characterized by the sharp separation of the individuals from the outside world and their great ability to react to external stimuli. It is very suggestive that this ability, at least so far as sight impressions are concerned, is developed to the utmost limit permitted by physics; for, as has often been remarked, only a few light quanta are needed to produce a visual sensation. Nevertheless, it is obviously a quite open question whether the information we have acquired of the laws describing atomic phenomena provides us with a sufficient basis for tackling the problem of living organisms, or whether, hidden behind the riddle of life, there lie yet unexplored aspects of epistemology.

Whatever the development in this domain may be, we have, as emphasized at the close of the article, every reason to rejoice that, within the relatively objective domain of physics, where emotional elements are so largely relegated to the background, we have encountered problems capable of reminding us anew of the general conditions underlying all human understanding, which, from time immemorial, have attracted the attention of philosophers.

Addendum (1931). The *fourth article*, which is an elaboration of a lecture delivered before the Scandinavian Meeting of Natural Scientists in 1929, is closely related to the other three articles, since it attempts to give a survey, against the same background, of the place of the atomic theory in the description of nature. In particular, it was my desire to emphasize that, despite the great success attending the discovery of the building stones of atoms—a discovery depending on the application of classical concepts—the development of the atomic theory has, nevertheless, first of all given us a recognition of laws which cannot be included within the frame formed by our accustomed modes of perception. As already indicated above, the lessons we have learned by the discovery of the quantum of action open up to us new prospects which may perhaps be of decisive importance, particularly in the discussion of the position of *living organisms* in our picture of the world.

If, according to the ordinary usage, we speak of a machine as dead, this only means that we can give a description, sufficient for our purpose, of its working in terms of the conceptual forms of classical mechanics. However, in view of the present recognition of the insufficiency of classical concepts in the domain of atomic theory this criterion of the inanimate is no longer suitable so far as atomic phenomena are concerned. Nevertheless, even the quantum mechanics may hardly depart sufficiently from the mode of description of classical mechanics to be capable of mastering the characteristic laws of life. In this connection, however, we must remember that the investigation of the phenomena of life not only leads us, as emphasized in the article, into that domain of atomic theory where the usual idealization of a sharp distinction between phenomena and their observation breaks down, but that, in addition, there is set a fundamental limit to the analysis of the phenomena of life in terms of physical concepts, since the interference necessitated by an observation which would be as complete as possible from the point of view of the atomic theory would cause the death of the organism. In other words: *the strict application of those concepts which are* *adapted to our description of inanimate nature might stand in a relationship of exclusion to the consideration of the laws of the phenomena of life.*

In exactly the same way as it is only possible on the basis of the fundamental complementarity between the applicability of the concept of atomic states and the co-ordination of the atomic particles in space and time to account, in a rational manner, for the characteristic stability of the properties of atoms, so might the peculiarity of the phenomena of life, and in particular the self-stabilizing power of organisms, be inseparably connected with the fundamental impossibility of a detailed analysis of the physical conditions under which life takes place. To put it briefly, one might perhaps say that quantum mechanics is concerned with the statistical behaviour of a given number of atoms under well-defined external conditions, while we are unable to define the state of a living being in terms of atomic measures; in fact, owing to the metabolism of the organism, it is not even possible to ascertain what atoms actually belong to the living individual. In this respect, the domain of the statistical quantum mechanics, which is based on the correspondence argument, occupies an intermediate position between the domain of applicability of the ideal of causal space-time mode of description and the domain of biology which is characterized by teleological arguments.

Although, put in the above way, this idea concerns only the physical aspect of the problem, it may perhaps also be suited to form a background for the ordering of the psychical aspects of life. As explained in the third article, and also touched upon above, the unavoidable influencing by introspection of all psychical experience, that is characterized by the feeling of volition, shows a striking similarity to the conditions responsible for the failure of causality in the analysis of atomic phenomena. Above all, as indicated there, an essential refinement of our interpretation, originally based on physical causality, of the psycho-physical parallelism ought to result from our taking into consideration the unpredictable modification of psychical experience produced by any attempt at an objective tracing of the accompanying physical processes in the central nervous system. With regard to this, however, it must not be forgotten that, in associating the psychical and physical aspects of existence, we are concerned with a special relationship of complementarity which it is not possible thoroughly to understand by one-sided application either of physical or of psychological laws. In consideration of the general lessons we have learned from the atomic theory, it would also seem likely that only a renunciation in this respect will enable us to comprehend, in the sense explained more fully in the fourth article, that harmony which is experienced as free will and analyzed in terms of causality.

I

Atomic Theory and Mechanics

(1925)

1. THE CLASSICAL THEORIES

The analysis of the equilibrium and the motion of bodies not only forms the foundation of physics, but for mathematical reasoning has also furnished a rich field, which has been exceedingly fertile for the development of the methods of pure mathematics. This connection between mechanics and mathematics showed itself at an early date in the works of Archimedes, Galilei and Newton. In their hands the formation of concepts suitable for the analysis of mechanical phenomena was provisionally completed. Since the time of Newton, the development of the methods for treating mechanical problems has gone hand in hand with the evolution of mathematical analysis; we need only recall such names as Euler, Lagrange and Laplace. The later development of mechanics too, based on the work of Hamilton, proceeded in very close association with the evolution of mathematical methods, the calculus of variations, and the theory of invariants, as appears clearly in recent times also in the papers of Poincaré.

Perhaps the greatest successes of mechanics lie in the domain of astronomy, but in the mechanical theory of heat an interesting application was also found in the course of the last century. The kinetic theory of gases, founded by Clausius and Maxwell, interprets the properties of gases to a large extent as results of the mechanical interactions of atoms and molecules flying about at random. We wish to recall especially the explanation of the two principles of thermodynamics given by this theory. The first principle is a direct result of the mechanical law of conservation of energy, while the second principle, the entropy law, can, following Boltzmann, be derived from the statistical behaviour of a large number of mechanical systems. It is of interest here that statistical considerations have permitted the description not only of the average behaviour of atoms, but also of the fluctuation phenomena, which have led by the investigation of the Brownian motion to the unexpected possibility of counting atoms. The proper tools for the systematic development of statistical mechanics, to which especially Gibbs contributed, were furnished by the mathematical theory of canonical systems of differential equations.

The development of the electromagnetic theories in the second half of last century, following the discoveries of Oersted and Faraday, brought about a profound generalization of mechanical concepts. Although, to begin with, mechanical models played an essential part in Maxwell's electrodynamics, the advantages were soon realized of conversely deriving the mechanical concepts from the theory of the electromagnetic field. In this theory the conservation laws are explained by considering energy and momentum to be localized in the space surrounding the bodies. In particular, a natural explanation of radiation phenomena can be obtained in this way. The theory of the electromagnetic field was the direct cause of the discovery of electromagnetic waves, which to-day play so important a part in electrical engineering. Further, the electromagnetic theory of light founded by Maxwell provided a rational basis for the wave theory of light, which goes back to Huygens. With the aid of the atomic theory, it afforded a general description of the origin of light and of the phenomena taking place during the passage of light through matter. For this purpose, the atoms are supposed to be built up of electrical particles which can execute vibrations about positions of equilibrium. The free oscillations of the particles are the cause of the radiation, the composition of which we observe in the atomic spectra of the elements. Further, the particles will execute forced vibrations under the forces in the light waves and thus become centres of secondary wavelets which will interfere with the primary waves and produce the well-known phenomena of reflection and refraction of light. When the frequency of vibration of the incident waves approaches the frequency of one of the free oscillations of the atom, there results a resonance effect, by which the particles are thrown into specially strong forced vibrations. In this way a natural account was obtained of the phenomena of resonance radiation and the anomalous dispersion of a substance for light near one of its spectral lines.

Just as in the kinetic theory of gases, it is not merely the average effect of a large number of atoms that comes into consideration in the electromagnetic interpretation of optical phenomena. Thus, in the scattering of light the random distribution of the atoms makes the effects of the individual atoms appear in such a way that a direct counting of the atoms is possible. In fact, Rayleigh estimated from the intensity of the scattered blue light of the sky the number of atoms in the atmosphere, obtaining results in satisfactory agreement with the counting of atoms obtained by Perrin from a study of the Brownian motion. The rational mathematical representation of the electromagnetic theory is based on the application of vector analysis, or more generally tensor analysis of higher dimensional manifolds. This analysis founded by Riemann offered the proper means for the formulation of Einstein's fundamental theory of relativity which introduces concepts that go beyond Galilei's kinematics and may perhaps be considered as the natural completion of the classical theories.

2. THE QUANTUM THEORY OF ATOMIC CONSTITUTION

In spite of all the successful applications of mechanical and electrodynamical ideas to atomic theory, further development revealed profound difficulties. If these theories really provide a general description of thermal agitation and of the radiation connected with motion, then the general laws of heat radiation must be capable of a direct explanation. Contrary to all expectations, however, a calculation on this basis could not explain the empirical laws. Going beyond this, Planck demonstrated, retaining Boltzmann's account of the second law of thermodynamics, that the laws of heat radiation demand an element of discontinuity in the description of atomic processes quite foreign to the classical theories. Planck discovered that in the statistical behaviour of particles which execute simple harmonic oscillations about positions of equilibrium only such states of vibrations must be taken into account the energy of which is an integral multiple of a "quantum", ωh, where ω is the frequency of the particle and h a universal constant, the so-called Planck's quantum of action.

The more precise formulation of the content of the quantum theory appears, however, to be extremely difficult when it is remembered that all the concepts of previous theories rest on pictures which demand the possibility of a continuous variation. This difficulty was especially emphasized by the fundamental researches of Einstein, according to which essential features of the interaction between light and matter suggest that the propagation of light does not take place by spreading waves but by "light-quanta", which, concentrated in a small region of space, contain the energy $h\nu$, where ν is the frequency of the light. The formal nature of this statement is evident because the definition and measurement of this frequency rests exclusively on the ideas of the wave theory.

The inadequacy of the classical theories was brought into prominence by the development of our knowledge of atomic structure. One formerly hoped that this knowledge might be gradually enlarged by an analysis of the properties of the elements based on the classical theories which had been fruitful in so many respects. This hope was supported shortly before the birth of the quantum theory by Zeeman's discovery of the effect of magnetic fields on spectral lines. As Lorentz showed, this effect corresponds in many cases to just that action of magnetic fields on the motion of oscillating particles which is to be expected from classical electrodynamics. Besides, this account allowed conclusions to be drawn about the nature of the oscillating particles which agreed beautifully with the experimental discoveries of Lenard and Thomson in the field of electric discharges in gases. As a result, small negatively charged particles, the electrons, were recognized as units common to all atoms. It is true that the so-called "anomalous" Zeeman effect of many spectral lines caused profound difficulties for the classical theory. These were similar to those which showed themselves in the attempts with the aid of electrodynamic models to explain the simple empirical regularities among the spectral frequencies which were brought to light through the work of Balmer, Rydberg, and Ritz. In particular, such an account of the spectral laws could scarcely be reconciled with the estimate of the number of electrons in the atom which Thomson obtained from observations on the scattering of X-rays by a direct application of the classical theory.

These difficulties could for a time be attributed to our imperfect knowledge of the origin of the forces by which the electrons are bound in the atom. The situation was, however, entirely changed by the experimental discoveries in the field of radioactivity, which furnished new means for the investigation of atomic structure. Thus, Rutherford obtained convincing support for the idea of the nuclear atom from experiments on the passage through matter of the particles ejected by radioactive substances. According to this idea, the greatest part of the atomic mass is localized in a positively charged nucleus, exceedingly small compared with the dimensions of the atom as a whole. Around the nucleus there move a number of light negative electrons. In this way, the problem of atomic structure took on a great similarity to the problems of celestial mechanics. A closer consideration, however, soon showed that, nevertheless, there exists a fundamental difference between an atom and a planetary system. The atom must have a stability which presents features quite foreign to mechanical theory. Thus, the mechanical laws permit a continuous variation of the possible motions, which is entirely at variance with the definiteness of the properties of the elements. The difference between an atom and an electrodynamic model appears also when one considers the composition of the emitted radiation. For, in models of the sort considered, where the natural frequencies of motion vary continuously with the energy, the frequency of the radiation will change continuously during emission according to classical theory and will therefore show no similarity to the line spectra of the elements.

The search for a more precise formulation of the concepts of the quantum theory which might be capable of overcoming these difficulties led to the enunciation of the following postulates:

(1) An atomic system possesses a certain manifold of states, the "stationary states", to which corresponds in general a discrete sequence of energy values and which have a peculiar stability. This latter shows itself in that every change in the energy of the atom must be due to a "transition" of the atom from one stationary state to another.

(2) The possibility of emission and absorption of radiation by the atom is conditioned by the possibility of energy changes of the atom, in such a way that the frequency of the radiation is connected with the energy difference between the initial and final states by the formal relation

$$h\nu = E_1 - E_2.$$

These postulates, which cannot be explained on classical ideas, seem to offer a suitable basis for the general account of the observed physical and chemical properties of the elements. In particular, an immediate explanation is given of a fundamental feature of the empirical spectral laws. This feature, the Ritz principle of combination of spectral lines, states that the frequency of every line in a spectrum can be represented as the difference between two terms of a manifold of spectral terms characteristic of the element; in fact, we see that these terms can be identified with the energy values of the stationary states of the atom, divided by h. In addition, this account of the origin of spectra gives an immediate explanation of the fundamental difference between absorption and emission spectra. For, according to the postulates, the condition for selective absorption of a frequency which corresponds to the combination of two terms is that the atom is in the state of smaller energy, while for emission of such radiation it must be in the state of greater energy. In short, the picture described is in very close agreement with the experimental results on the excitation of spectra. This is shown especially in the discovery of Franck and Hertz as to impacts between free electrons and atoms. They found that an energy transfer from the electron to the atom can take place only in amounts which are just the energy differences of the stationary states as computed from the spectral terms. In general, the atom is simultaneously excited to emit. Similarly, the excited atom can, according to Klein and Rosseland, lose its emissive power through an impact, and the colliding electron experiences a corresponding increase of its energy.

As Einstein has shown, the postulates also furnish a suitable basis for a rational treatment of statistical problems, especially for a very lucid derivation of Planck's law of radiation. This theory assumes that an atom, which can undergo a transition between two stationary states and is in the higher state, has a certain "probability", depending only on the atom, of jumping spontaneously to the lower state in a given interval of time. Further, it assumes that external illumination with radiation of the frequency corresponding to the transition gives the atom a probability, proportional to the intensity of the radiation, of going from the lower to the higher state. It is also an essential feature of the theory that illumination with this frequency gives the atom in the higher state, besides its spontaneous probability, an induced probability of jumping down to the lower state.

While Einstein's theory of heat radiation gives support to the postulates, it accentuates the formal nature of the frequency condition. For, from the conditions for complete thermal equilibrium, Einstein draws the conclusion that every absorption and emission process is accompanied by a transfer of momentum of amount $h\nu/c$, where c is the velocity of light, just as the idea of light quanta would lead one to expect The significance of this conclusion has been emphasized in a very interesting way by the discovery of Compton that the scattering of homogeneous X-rays is accompanied by a change of wavelength in the scattered radiation depending on the direction of observation. Such a change in frequency follows in a simple way from the light-quantum theory if in the deflection of the quantum one takes into account the conservation of momentum as well as energy.

The constantly growing contrast between the wave theory of light, apparently required for the explanation of optical phenomena, and the light-quantum theory, which represents naturally so many features of the interaction between light and matter, suggested that the failure of classical theories may even affect the validity of the laws of conservation of energy and momentum. These laws, which hold so central a position in the classical theory, would, then, in the description of atomic processes, be only statistically valid. However, this suggestion does not offer a satisfactory escape from the dilemma, as is shown by the experiments on the scattering of X-rays which have been undertaken recently with the beautiful methods permitting a direct observation of individual processes. For Geiger and Bothe have been able to show that the recoil electrons and photo-electrons which accompany the production and absorption of the scattered radiation are coupled in pairs just as one would expect from the picture of the light-quantum theory. With the method of the Wilson cloud chamber, Compton and Simon have even succeeded in demonstrating, besides this pairing, the connection demanded by the light-quantum theory between the direction in which the effect of the scattered radiation is observed and the direction of the velocity of the recoil electrons accompanying the scattering.

From these results it seems to follow that, in the general problem of the quantum theory, one is faced not with a modification of the mechanical and electrodynamical theories describable in terms of the usual physical concepts, but with an essential failure of the pictures in space and time on which the description of natural phenomena has hitherto been based. This failure appears also in a closer consideration of impact phenomena. In particular, for impacts in which the time of collision is short compared to the natural periods of the atom and for which very simple results are to be expected according to the usual mechanical ideas, the postulate of stationary states would seem to be irreconcilable with any description of the collision in space and time based on the accepted ideas of atomic structure.

3. THE CORRESPONDENCE PRINCIPLE

Nevertheless, it has been possible to construct mechanical pictures of the stationary states which rest on the concept of the nuclear atom and have been essential in interpreting the specific properties of the elements. In the simplest case of an atom with only one electron, such as the neutral hydrogen atom, the orbit of the electron would be in classical mechanics a closed ellipse, obeying Kepler's laws, according to which the major axis and frequency of revolution are connected in a simple way with the work necessary for a complete separation of the atomic particles. Now if we regard the spectral terms of the hydrogen spectrum as determining this work, we see in that spectrum evidence of the steplike process through which the electron under emission of radiation is gradually bound more and more firmly in states visualized as orbits of smaller and smaller dimensions. When the electron is bound as firmly as possible, and the atom can therefore emit no further radiation, the normal state of the atom has been reached. The orbital dimensions estimated from the spectral terms have values for this state of the same order of magnitude as the atomic dimensions obtained from the mechanical properties of the elements. From the nature of the postulates, however, such features of the mechanical pictures as frequency of revolution and shape of the electronic orbits are not open to comparison with observations. The symbolic character of these pictures can scarcely be more strongly emphasized than by the fact that in the normal state no radiation is emitted, although according to the mechanical picture the electron is still moving.

Nevertheless, the visualization of the stationary states by mechanical pictures has brought to light a far-reaching analogy between the quantum theory and the mechanical theory. This analogy was traced by investigating the conditions in the initial stages of the binding process described, where the motions corresponding to successive stationary states differ comparatively little from each other. Here it was possible to demonstrate an asymptotic agreement between spectrum and motion. This agreement establishes a quantitative relation by which the constant appearing in Balmer's formula for the hydrogen spectrum is expressed in terms of Planck's constant and the values of the charge and mass of the electron. The essential validity of this relation was clearly illustrated by the subsequent test of the predictions of the theory regarding the dependence of the spectrum on the nuclear charge. The latter result may be considered as the first step towards the fulfilment of the programme presented by the concept of the nuclear atom, to account for the relationships between the properties of the elements solely by means of the integer which represents the number of unit charges on the nucleus, the so-called "atomic number".

The demonstration of the asymptotic agreement between spectrum and motion gave rise to the formulation of the "correspondence principle", according to which the possibility of every transition process connected with emission of radiation is conditioned by the presence of a corresponding harmonic component in the motion of the atom. Not only do the frequencies of the corresponding harmonic components agree asymptotically with the values obtained from the frequency condition in the limit where the energies of the stationary states converge, but also the amplitudes of the mechanical oscillatory components give in this limit an asymptotic measure for the probabilities of the transition processes on which the intensities of the observable spectral lines depend. The correspondence principle expresses the tendency to utilize in the systematic development of the quantum theory every feature of the classical theories in a rational transcription appropriate to the fundamental contrast between the postulates and the classical theories.

The development was considerably furthered by the fact that it seemed possible to formulate certain general laws, the so-called rules of "quantization", by means of which the mechanical motions associated with the stationary states were to be chosen from the continuous manifold of such motions. These rules concern atomic systems for which the solution of the mechanical equations of motion is simply- or multiply-periodic. In these cases the motion of every particle can be represented as a superposition of discrete harmonic vibrations. According to the rules of quantization which were considered to be a rational generalization of Planck's original result for the possible energy values of an harmonic oscillator, certain components of action which characterize the solution of the mechanical equations of motion are put equal to integral multiples of Planck's constant. By means of these rules a classification of the stationary states results in which a set of integers, the "quantum indices", is ascribed to every state. Their number is equal to the degree of periodicity of the mechanical motion.

In formulating the rules of quantization, the modern development of the mathematical methods of dealing with mechanical problems was of decisive importance. We need recall only the theory of phase integrals, utilized in particular by Sommerfeld, as well as the property of adiabatic invariance of these integrals emphasized by Ehrenfest. The theory was given a very elegant form by the introduction of the uniformizing variables of Stäckel. In this form the fundamental frequencies determining the periodicity properties of the mechanical solution appear as the partial derivatives of the energy with respect to the components of action to be quantized. The asymptotic connection between the motion and the spectrum as calculated from the frequency condition is secured in this way.

With the help of the rules of quantization, many finer details of spectra seemed to be accounted for naturally. Of especial interest was Sommerfeld's demonstration that the small deviations from a Keplerian motion, which result from the modification of Newtonian mechanics demanded by the relativity theory, offer an explanation of the fine structure of the hydrogen lines. Furthermore, we would recall here the explanation given by Epstein and Schwarzschild of the splitting up of the hydrogen lines in an external electric field which Stark discovered.

We are here dealing with a mechanical problem, the treatment of which was much improved in the hands of mathematicians like Euler and Lagrange, until Jacobi stated his famous elegant solution by means of Hamilton's partial differential equation. Especially after the utilization of the correspondence principle—by which not only the polarization of the Stark effect components was interpreted, but also, as Kramers showed, the peculiar intensity distribution of these components—can we say that in this effect every trait of Jacobi's solution can be recognized, although hidden under a quantum theory mask. In this connection it is of interest to mention that with the help of the correspondence principle, the effect of a magnetic field on the hydrogen atom could be treated so as to show a far-reaching similarity with Lorentz's account of the Zeeman effect on the basis of classical electrodynamics, especially in the form given by Larmor.

4. RELATIONSHIPS BETWEEN THE ELEMENTS

While the last-mentioned problems represent direct applications of the rules of quantization, we meet, in the problem of the structure of atoms with several electrons, a case where the general solution of the mechanical problem does not possess the periodic properties which seem to be necessary for the mechanical picturing of the stationary states. It suggested itself, however, that this further limitation of the applicability of mechanical pictures in the study of the properties of atoms with several electrons, beyond that in the study of atoms containing only one electron, is directly connected with the postulate of the stability of stationary states. In fact, the interaction of the electrons in the atom presents a problem which is quite analogous to the problem of a collision between an atom and a free electron. Just as no mechanical explanation can be given for the stability of an atom in the collision, so we must suppose that already, in the description of the stationary states of the atom, the special part which every electron plays in its interaction with the other electrons is secured in an entirely unmechanical way.

This view is in general conformity with the spectroscopic evidence. An important feature of this evidence is the discovery of Rydberg that, in spite of the more complicated structure of the spectra of other elements compared to that of hydrogen, the same constant as that in the Balmer formula appears in the empirical formulae of the series spectra of all elements. This discovery is simply explained by regarding the series spectra as evidence of processes by which an electron is added to an atom, its binding becoming more firm step by step with the emission of radiation. While the character of the binding of the other electrons remains the same, the steplike strengthening of the binding of this electron is visualized by orbits which at first are large compared with usual atomic dimensions, and become smaller and smaller until the normal state of the atom is reached. In the case when the atom has a single positive charge before the capture of the electron, the attraction for the electron by the rest of the atom will, on this picture of the binding process, at first coincide closely with the attraction of the particles in the hydrogen atom. It is therefore clear why the spectral terms representing the binding of the electrons show an asymptotic convergence to the terms of the hydrogen spectrum. In the same way one obtains a direct explanation of the general dependence of a series spectrum on the state of ionization of the atom, brought to light so beautifully through the work of Fowler and Paschen.

Typical evidence of the way in which the electrons are bound in the atoms is also afforded by the study of the X-ray spectra. On one hand, the fundamental discovery of Moseley, of the striking similarity of the X-ray spectrum of an element to the spectrum that corresponds to the binding of a single electron by the nucleus, can be easily understood if it is remembered that in the interior of the atom the direct influence of the nucleus on the nature of the binding of each individual electron exceeds greatly the mutual influence of the electrons. On the other hand, the X-ray spectra show certain characteristic differences from the series spectra. These originate from the circumstance that in the former we do not witness the binding of an additional electron in the atom, but the reorganization of the binding of the remaining electrons upon removal of one of the electrons previously bound. This circumstance, which has been especially emphasized by Kossel, was well suited for bringing to light new and important features of the stability of atomic structure.

To account for the details of the spectra, a closer study of the interplay of the electrons in the atom is, of course, required. Disregarding a strict application of mechanics, an attack was made on this problem by assigning to every electron a motion of such periodic properties that a classification of the spectral terms by means of quantum indices could be accomplished. In the hands of Sommerfeld, in particular, a number of regularities of spectra were simply explained in this way. Further, these considerations afforded a fruitful field of application for the correspondence principle. In fact, this principle could explain the peculiar limitations in the possibilities of combining spectral terms, the so-called selection rules for spectral lines.

On these lines, it has recently been possible by making use of the evidence from series spectra, as well as from X-ray spectra, to draw conclusions about the grouping of electrons in the normal state of the atom. This grouping explained the general features of the periodic system of the elements in conformity with the ideas of chemical activity of atoms as developed especially by J. J. Thomson, Kossel, and G. N. Lewis. Progress in this field has been intimately connected with the great enrichment during the last few years of spectroscopic evidence, and, not least, by the investigations of Lyman and Millikan, the gap has been almost bridged over between the optical spectra and the region of X-rays, where great advances have been made in recent years by Siegbahn and his collaborators. In this connection the work of Coster on the X-ray spectra of heavy elements may be mentioned as affording beautiful support for the account of essential features of the periodic system.

5. INSUFFICIENCY OF MECHANICAL PICTURES

The analysis of the finer details of the spectra, however, has brought to light a number of features which could not be interpreted with mechanical pictures on the basis of the theory of periodicity systems. We refer in particular to the multiplet structure of spectral lines and the effect of magnetic fields on these structures. These latter phenomena, which are generally known as anomalous Zeeman effects and, as mentioned above, had already led to difficulties in the classical theories, fitted, it is true, in a natural way into the scheme of the fundamental postulates of the quantum theory. For, as Landé showed, the frequencies of the components into which each spectral line is split up by the field can, like the original lines, be represented as combinations of terms. The manifold of these magnetic terms is obtained by replacing each original spectral term by a set of values which differ from it by small quantities depending on the field intensity. In fact, the beautiful experiments of Stern and Gerlach, by which a direct connection was established between the force which acts on an atom in a non-homogeneous magnetic field and the energy values of the stationary states in the field calculated from the magnetic terms, may be regarded as a most direct support of the fundamental ideas of the quantum theory.

Landé's analysis discloses, however, a strange difference between the interactions of the electrons in the atom and the coupling of mechanical systems. In fact, we are forced to assume the presence of a mechanically undescribable "strain" in the interaction of the electrons which prevents a unique assignment of quantum indices on the basis of mechanical pictures. In the discussion of this problem a general condition of thermodynamic stability introduced by Ehrenfest played an important part. When applied to the postulates of the quantum theory, this condition states that the statistical weight attributed to a stationary state is a quantity which cannot be changed by a continuous transformation of the atomic system. Moreover, it has recently been recognized that this same condition leads, even for atoms with only one electron, to difficulties which point to a limitation of the validity of the theory of periodicity systems. In fact, the problem of the motion of point charges admits of certain singular solutions which must be excluded from the manifold of stationary states. This exclusion artificially restricts the rules of quantization; but at first this restriction did not obviously contradict experimental evidence. Difficulties of an especially grave nature, however, were brought to light by the interesting analysis by Klein and Lenz of the problem of a hydrogen atom in crossed electric and magnetic fields. Here it was found impossible to satisfy Ehrenfest's condition, since suitable variation of the external forces would gradually transform orbits depicting stationary states which could not always be excluded from the manifold of these states into orbits where the electron falls into the nucleus.

Apart from these difficulties, the analysis of the finer details of the spectra has considerably furthered the quantum theory interpretation of the laws of the relationship between the elements. In fact, an extension of the ideas regarding the grouping of the electrons to which the quantum theory has led has recently been suggested by Dauvillier, Main Smith, and Stoner, taking various kinds of evidence into consideration. Notwithstanding the formal nature of these suggestions, they exhibit a close connection with the spectral regularities disclosed by Landé's analysis. In this direction important progress has recently been made, especially by Pauli. Notwithstanding that the results thus obtained constitute an important step towards the above-mentioned programme of accounting for the properties of the elements solely on the basis of the atomic number, it must be remembered, however, that the results do not allow of a unique association with mechanical pictures.

A new era in the development of the quantum theory has been opened up during the last few years by a closer study of optical phenomena. While, as mentioned above, the classical theory had such great successes in this field, the postulates at first gave no direct clue. From experiment, it is true, one could conclude that an atom, when illuminated, caused a scattering of the light essentially analogous to the classically computed scattering of elastically bound electrical particles, the natural frequencies of which are the same as the frequencies corresponding to the transition processes which the atom can perform under the influence of external radiation. In fact, on the classical theory such harmonic oscillators would, when excited, emit a radiation of just the same constitution as that of atoms transferred to a higher stationary state.

The possibility of obtaining a unified description of optical phenomena with this concept of oscillators conjugated to the transitions was essentially advanced by an idea of Slater, according to which the emission of radiation from an activated atom may be regarded as "cause" for the spontaneous transitions, in analogy to the effect of incident radiation in producing transitions. Ladenburg made a first important step towards a quantitative description of dispersion by suggesting a definite connection between the scattering activity of the oscillators and the probabilities of the corresponding transitions in Einstein's theory. Decisive progress, however, was made by Kramers by an ingenious transcription in harmony with the correspondence principle of the effects which, according to classical theory, are brought about in an electrodynamical system by illumination with light waves. Just as the radiation frequencies are calculated in the classical theory, on one hand, and the quantum theory on the other hand, it is typical in this transcription, that differential quotients are replaced by differences, in such a way that in the final formulae only quantities open to direct observation appear. Thus in Kramers' theory the scattering of an atom in a certain stationary state is quantitatively connected with the frequencies corresponding to the different transition processes to other stationary states as well as to the probabilities of the appearance of these transitions under the influence of illumination.

It is an essential feature of the theory that in calculating the anomalous dispersion near a spectral line, one has to take into account two opposite kinds of resonance effects depending on whether the spectral line corresponds to a transition of the atom to a state of larger or smaller energy. Only the first of these corresponds with the resonance effects which have been utilized previously in accounting for dispersion on the basis of the classical theory. It is also very interesting that the further development of the theory by Kramers and Heisenberg gave a natural quantitative description of additional scattering effects with changed frequencies, the existence of which had been predicted by Smekal from considerations based on the theory of light quanta, which thereby again has shown its fertility.

While this description of optical phenomena was entirely in harmony with the fundamental ideas of the quantum theory, it soon appeared that it stood in strange contradiction to the use of the mechanical pictures previously employed for an analysis of the stationary states. In the first place, it is impossible on the basis of the scattering activity of illuminated atoms demanded by the dispersion theory to construct an asymptotic connection between the reaction of an atom in alternating fields of smaller and smaller frequency and the reaction in constant fields as calculated from quantization rules of the theory of periodicity systems. This difficulty strengthened the doubts about this theory to which, as already mentioned, the problem of the hydrogen atom in crossed electric and magnetic fields had led. Secondly, it had to be regarded as especially unsatisfactory that the theory of periodicity systems was apparently helpless in the problem of the quantitative determination of the transition probabilities on the basis of the mechanical pictures of stationary states. This was felt all the more, as it was possible in several cases to obtain a quantitative formulation of the general statements of the correspondence principle as regards these transition probabilities with the help of viewpoints suggested by an analysis of the optical behaviour of electrodynamic models. These results stood in excellent agreement with measurements on the relative intensities of spectral lines, as they have been developed especially in Utrecht during the last few years, but they could only in a very artificial way be included in the schemes governed by the rules of quantization.

6. THE DEVELOPMENT OF A RATIONAL QUANTUM MECHANICS

Quite recently Heisenberg, who has especially emphasized these difficulties, has taken a step probably of fundamental importance by formulating the problems of the quantum theory in a novel way by which the difficulties attached to the use of mechanical pictures may, it is hoped, be avoided. In this theory the attempt is made to transcribe every use of mechanical concepts in a way suited to the nature of the quantum theory, and such that in every stage of the computation only directly observable quantities enter. In contrast to ordinary mechanics, the new quantum mechanics does not deal with a space-time description of the motion of atomic particles. It operates with manifolds of quantities which replace the harmonic oscillating components of the motion and symbolize the possibilities of transitions between stationary states in conformity with the correspondence principle. These quantities satisfy certain relations which take the place of the mechanical equations of motion and the quantization rules.

That such a procedure actually leads to a self-contained theory sufficiently analogous to classical mechanics depends essentially on the fact that, as Born and Jordan were able to show, there exists in Heisenberg's quantum mechanics a conservation theorem analogous to the energy law of classical mechanics. The theory is built up in such a way that it is automatically in harmony with the postulates of the quantum theory. In particular, the frequency condition is fulfilled by the values for energies and frequencies derived from the quantum mechanical equations of motion. Although the fundamental relations which take the place of the quantization rules involve Planck's constant, quantum indices do not appear explicitly in these relations. The classification of stationary states is based solely on a consideration of the transition possibilities, which enable the manifold of these states to be built up step by step. In brief, the whole apparatus of the quantum mechanics can be regarded as a precise formulation of the tendencies embodied in the correspondence principle. It must here be mentioned that the theory fulfils the requirements of Kramers' dispersion theory.

Owing to the great difficulties of the mathematical problem involved, it has not yet been possible to apply Heisenberg's theory to questions of atomic structure. From the above brief description it will be understood, however, that a number of results, which, like the expression for Rydberg's constant, had formerly been obtained on the basis of mechanical pictures by the aid of the correspondence principle, will retain their validity. Moreover, it is of the greatest interest that already in the simple cases for which up to now a treatment on the basis of Heisenberg's theory has been carried out, the new theory leads, besides, to a quantitative calculation of the transition probabilities and to energy values for the stationary states which differ systematically from those obtained by the quantization rules of the older theory. One may therefore hope that Heisenberg's theory will be helpful in the struggle with the puzzling difficulties, mentioned above, in the study of the finer details of the spectra.

Earlier in this paper mention was made of the fundamental difficulties involved in the construction of pictures of the interaction between atoms either by means of radiation or by collisions. These difficulties seem to require just that renunciation of mechanical models in space and time which is so characteristic a feature in the new quantum mechanics. As yet, however, the formulation of this mechanics takes no account of the coupling of transition processes in pairs which shows itself in those interactions. In fact, only those quantities which depend on the existence of the stationary states and the possibilities of transitions between them occur in the new theory, which definitely avoids any mention of the times at which transitions take place. This restriction, however, which is typical of the attack on the problem of the constitution of the atom based on the postulates of the quantum theory, allows only some aspects of the analogy between the quantum theory and the classical theories to come to light. These aspects belong principally to the radiative properties of atoms, and just here Heisenberg's theory represents a real advance. In particular it allows us, in the phenomena of scattering, to recognize the presence of electrons bound in atoms in a way completely analogous to the classical theories, which, as stated above, in the hands of J. J. Thomson enabled the number of electrons in an atom to be counted from measurements of scattering of X-rays. The problems arising out of the validity of the conservation laws in atomic interaction involve, however, quite other aspects of the correspondence of the quantum theory with the classical theory. These are equally essential in the general formulation of the quantum theory, and it is impossible to avoid discussing them when the reaction of atoms to swiftly moving particles is more closely studied. It is just here, indeed, that the classical theories have contributed so fundamentally to our knowledge of atomic structure.

It will interest mathematical circles that the mathematical instruments created by the higher algebra play an essential part in the rational formulation of the new quantum mechanics. Thus, the general proofs of the conservation theorems in Heisenberg's theory carried out by Born and Jordan are based on the use of the theory of matrices, which go back to Cayley and were developed especially by Hermite. It is to be hoped that a new era of mutual stimulation of mechanics and mathematics has commenced. To the physicists it will at first seem deplorable that in atomic problems we have apparently met with such a limitation of our usual means of visualization. This regret will, however, have to give way to thankfulness that mathematics in this field, too, presents us with the tools to prepare the way for further progress.

II

The Quantum Postulate and the Recent Development of Atomic Theory

(1927)

Although it is with great pleasure that I follow the kind invitation of the presidency of the congress to give an account of the present state of the quantum theory in order to open a general discussion on this subject, which takes so central a position in modern physical science, it is with a certain hesitation that I enter on this task. Not only is the venerated originator of the theory present himself, but among the audience there will be several who, due to their participation in the remarkable recent development, will surely be more conversant with details of the highly developed formalism than I am. Still I shall try, by making use only of simple considerations and without going into any details of technical mathematical character, to describe to you a certain general point of view which I believe is suited to give an impression of the general trend of the development of the theory from its very beginning and which I hope will be helpful in order to harmonize the apparently conflicting views taken by different scientists. No subject indeed may be better suited than the quantum theory to mark the development of physics in the century passed since the death of the great genius, whom we are here assembled to commemorate. At the same time, just in a field like this where we are wandering on new paths and have to rely upon our own judgment in order to escape from the pitfalls surrounding us on all sides, we have perhaps more occasion than ever at every step to be remindful of the work of the old masters who have prepared the ground and furnished us with our tools.

1. QUANTUM POSTULATE AND CAUSALITY

The quantum theory is characterized by the acknowledgment of a fundamental limitation in the classical physical ideas when applied to atomic phenomena. The situation thus created is of a peculiar nature, since our interpretation of the experimental material rests essentially upon the classical concepts. Notwithstanding the difficulties which, hence, are involved in the formulation of the quantum theory, it seems, as we shall see, that its essence may be expressed in the so-called quantum postulate, which attributes to any atomic process an essential discontinuity, or rather individuality, completely foreign to the classical theories and symbolized by Planck's quantum of action.

This postulate implies a renunciation as regards the causal space-time co-ordination of atomic processes. Indeed, our usual description of physical phenomena is based entirely on the idea that the phenomena concerned may be observed without disturbing them appreciably. This appears, for example, clearly in the theory of relativity, which has been so fruitful for the elucidation of the classical theories. As emphasized by Einstein, every observation or measurement ultimately rests on the coincidence of two independent events at the same space-time point. Just these coincidences will not be affected by any differences which the space-time co-ordination of different observers otherwise may exhibit. Now, the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected. Accordingly, an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation. After all, the concept of observation is in so far arbitrary as it depends upon which objects are included in the system to be observed. Ultimately, every observation can, of course, be reduced to our sense perceptions. The circumstance, however, that in interpreting observations use has always to be made of theoretical notions entails that for every particular case it is a question of convenience at which point the concept of observation involving the quantum postulate with its inherent "irrationality" is brought in.

This situation has far-reaching consequences. On one hand, the definition of the state of a physical system, as ordinarily understood, claims the elimination of all external disturbances. But in that case, according to the quantum postulate, any observation will be impossible, and, above all, the concepts of space and time lose their immediate sense. On the other hand, if in order to make observation possible we permit certain interactions with suitable agencies of measurement, not belonging to the system, an unambiguous definition of the state of the system is naturally no longer possible, and there can be no question of causality in the ordinary sense of the word. The very nature of the quantum theory thus forces us to regard the space-time co-ordination and the claim of causality, the union of which characterizes the classical theories, as complementary but exclusive features of the description, symbolizing the idealization of observation and definition respectively. Just as the relativity theory has taught us that the convenience of distinguishing sharply between space and time rests solely on the smallness of the velocities ordinarily met with compared to the velocity of light, we learn from the quantum theory that the appropriateness of our usual causal space-time description depends entirely upon the small value of the quantum of action as compared to the actions involved in ordinary sense perceptions. Indeed, in the description of atomic phenomena, the quantum postulate presents us with the task of developing a "complementarity" theory the consistency of which can be judged only by weighing the possibilities of definition and observation.

This view is already clearly brought out by the much-discussed question of the nature of light and the ultimate constituents of matter. As regards light, its propagation in space and time is adequately expressed by the electromagnetic theory. Especially the interference phenomena *in vacuo* and the optical properties of material media are completely governed by the wave theory superposition principle. Nevertheless, the conservation of energy and momentum during the interaction between radiation and matter, as evident in the photo-electric and Compton effect, finds its adequate expression just in the light quantum idea put forward by Einstein. As is well known, the doubts regarding the validity of the superposition principle, on one hand, and of the conservation laws, on the other, which were suggested by this apparent contradiction, have been definitely disproved through direct experiments. This situation would seem clearly to indicate the impossibility of a causal space-time description of the light phenomena. On one hand, in attempting to trace the laws of the time-spatial propagation of light according to the quantum postulate, we are confined to statistical considerations. On the other hand, the fulfilment of the claim of causality for the individual light processes, characterized by the quantum of action, entails a renunciation as regards the space-time description. Qf course, there can be no question of a quite independent application of the ideas of space and time and of causality. The two views of the nature of light are rather to be considered as different attempts at an interpretation of experimental evidence in which the limitation of the classical concepts is expressed in complementary ways.

The problem of the nature of the constituents of matter presents us with an analogous situation. The individuality of the elementary electrical corpuscles is forced upon us by general evidence. Nevertheless, recent experience, above all the discovery of the selective reflection of electrons from metal crystals, requires the use of the wave theory superposition principle in accordance with the original ideas of L. de Broglie. Just as in the case of light, we have consequently in the question of the nature of matter, so far as we adhere to classical concepts, to face an inevitable dilemma which has to be regarded as the very expression of experimental evidence. In fact, here again we are not dealing with contradictory but with complementary pictures of the phenomena, which only together offer a natural generalization of the classical mode of description. In the discussion of these questions, it must be kept in mind that, according to the view taken above, radiation in free space as well as isolated material particles are abstractions, their properties on the quantum theory being definable and observable only through their interaction with other systems. Nevertheless, these abstractions are, as we shall see, indispensable for a description of experience in connection with our ordinary space-time view.

The difficulties with which a causal space-time description is confronted in the quantum theory, and which have been the subject of repeated discussions, are now placed into the foreground by the recent development of the symbolic methods. An important contribution to the problem of a consistent application of these methods has been made lately by Heisenberg. In particular, he has stressed the peculiar reciprocal uncertainty which affects all measurements of atomic quantities. Before we enter upon his results, it will be advantageous to show how the complementary nature of the description appearing in this uncertainty is unavoidable already in an analysis of the most elementary concepts employed in interpreting experience.

2. QUANTUM OF ACTION AND KINEMATICS

The fundamental contrast between the quantum of action and the classical concepts is immediately apparent from the simple formulae which form the common foundation of the theory of light quanta and of the wave theory of material particles. If Planck's constant be denoted by h, as is well known,

$$E\tau = I\lambda = h, \qquad \ldots\ldots(1)$$

where E and I are energy and momentum respectively, τ and λ the corresponding period of vibration and wave-length. In these formulae the two notions of light and also of matter enter in sharp contrast. While energy and momentum are associated with the concept of particles, and, hence, may be characterized according to the classical point of view by definite space-time co-ordinates, the period of vibration and wave-length refer to a plane harmonic wave train of unlimited extent in space and time. Only with the aid of the superposition principle does it become possible to attain a connection with the ordinary mode of description. Indeed, a limitation of the extent of the wave-fields in space and time can always be regarded as resulting from the interference of a group of elementary harmonic waves. As shown by de Broglie, the translational velocity of the individuals associated with the waves can be represented by just the so-called group-velocity. Let us denote a plane elementary wave by

$$A \cos 2\pi\, (\nu t - x\sigma_x - y\sigma_y - z\sigma_z + \delta),$$

where A and δ are constants determining respectively the amplitude and the phase. The quantity $\nu = 1/\tau$ is the frequency, σ_x, σ_y, σ_z the wave numbers in the direction of the co-ordinate axes, which may be regarded as vector components of the wave number $\sigma = 1/\lambda$ in the directions of propagation. While the wave or phase velocity is given by ν/σ, the group-velocity is defined by $d\nu/d\sigma$. Now according to the relativity theory we have for a particle with the velocity v:

$$I = \frac{v}{c^2} E \text{ and } v dI = dE,$$

where c denotes the velocity of light. Hence by equation (1) the phase velocity is c^2/v and the group-velocity v. The circumstance that the former is in general greater than the velocity of light emphasizes the symbolic character of these considerations. At the same time, the possibility of identifying the velocity of the particle with the group-velocity indicates the field of application of space-time pictures in the quantum theory. Here the complementary character of the description appears, since the use of wave-groups is necessarily accompanied by a lack of sharpness in the definition of period and wave-length, and hence also in the definition of the corresponding energy and momentum as given by relation (1).

Rigorously speaking, a limited wave-field can only be obtained by the superposition of a manifold of elementary waves corresponding to all values of ν and σ_x, σ_y, σ_z. But the order of magnitude of the mean difference between these values for two elementary waves in the group is given in the most favourable case by the condition

$$\Delta t\,\Delta\nu = \Delta x\,\Delta\sigma_x = \Delta y\,\Delta\sigma_y = \Delta z\,\Delta\sigma_z = 1,$$

where Δt, Δx, Δy, Δz denote the extension of the wave-field in time and in the directions of space corresponding to the co-ordinate axes. These relations—well known from the theory of optical instruments, especially from Rayleigh's investigation of the resolving power of spectral apparatus—express the condition that the wave-trains extinguish each other by interference at the space-time boundary of the wave-field. They may be regarded also as signifying that the group as a whole has no phase in the same sense as the elementary waves. From equation (1) we find thus:

$$\Delta t\,\Delta E = \Delta x\,\Delta I_x = \Delta y\,\Delta I_y = \Delta z\,\Delta I_z = h, \quad \ldots\ldots(2)$$

as determining the highest possible accuracy in the definition of the energy and momentum of the individuals associated with the wave-field. In general, the conditions for attributing an energy and a momentum value to a wave-field by means of formula (1) are much less favourable. Even if the composition of the wave-group corresponds in the beginning to the relations (2), it will in the course of time be subject to such changes that it becomes less and less suitable for representing an individual. It is this very circumstance which gives rise to the paradoxical character of the problem of the nature of light and of material particles. The limitation in the classical concepts expressed through relation (2), is, besides, closely connected with the limited validity of classical mechanics, which in the wave theory of matter corresponds to the geometrical optics in which the propagation of waves is depicted through "rays". Only in this limit can energy and momentum be unambiguously defined on the basis of space-time pictures. For a general definition of these concepts we are confined to the conservation laws, the rational formulation of which has been a fundamental problem for the symbolical methods to be mentioned below.

In the language of the relativity theory, the content of the relations (2) may be summarized in the statement that according to the quantum theory a general reciprocal relation exists between the maximum sharpness of definition of the space-time and energy-momentum vectors associated with the individuals. This circumstance may be regarded as a simple symbolical expression for the complementary nature of the space-time description and the claims of causality. At the same time, however, the general character of this relation makes it possible to a certain extent to reconcile the conservation laws with the space-time co-ordination of observations, the idea of a coincidence of well-defined events in a space-time point being replaced by that of unsharply defined individuals within finite space-time regions.

This circumstance permits us to avoid the well-known paradoxes which are encountered in attempting to describe the scattering of radiation by free electrical particles as well as the collision of two such particles. According to the classical concepts, the description of the scattering requires a finite extent of the radiation in space and time, while in the change of the motion of the electron demanded by the quantum postulate one seemingly is dealing with an instantaneous effect taking place at a definite point in space. Just as in the case of radiation, however, it is impossible to define momentum and energy for an electron without considering a finite space-time region. Furthermore, an application of the conservation laws to the process implies that the accuracy of definition of the energy-momentum vector is the same for the radiation and the electron. In consequence, according to relation (2), the associated space-time regions can be given the same size for both individuals in interaction.

A similar remark applies to the collision between two material particles, although the significance of the quantum postulate for this phenomenon was disregarded before the necessity of the wave concept was realized. Here, this postulate does, indeed, represent the idea of the individuality of the particles which, transcending the space-time description, meets the claim of causality. While the physical content of the light-quantum idea is wholly connected with the conservation theorems for energy and momentum, in the case of the electrical particles the electric charge has to be taken into account in this connection. It is scarcely necessary to mention that for a more detailed description of the interaction between individuals we cannot restrict ourselves to the facts expressed by formulae (1) and (2), but must resort to a procedure which allows us to take into account the coupling of the individuals, characterizing the interaction in question, where just the importance of the electric charge appears. As we shall see, such a procedure necessitates a further departure from visualization in the usual sense.

3. MEASUREMENTS IN THE QUANTUM THEORY

In his investigations already mentioned on the consistency of the quantum-theoretical methods, Heisenberg has given the relation (2) as an expression for the maximum precision with which the space-time coordinates and momentum-energy components of a particle can be measured simultaneously. His view was based on the following consideration: On one hand, the co-ordinates of a particle can be measured with any desired degree of accuracy by using, for example, an optical instrument, provided radiation of sufficiently short wave-length is used for illumination. According to the quantum theory, however, the scattering of radiation from the object is always connected with a finite change in momentum, which is the larger the smaller the wave-length of the radiation used. The momentum of a particle, on the other hand, can be determined with any desired degree of accuracy by measuring, for example, the Doppler effect of the scattered radiation, provided the wave-length of the radiation is so large that the effect of recoil can be neglected, but then the determination of the space co-ordinates of the particle becomes correspondingly less accurate.

The essence of this consideration is the inevitability of the quantum postulate in the estimation of the possibilities of measurement. A closer investigation of the possibilities of definition would still seem necessary in order to bring out the general complementary character of the description. Indeed, a discontinous change of energy and momentum during observation could not prevent us from ascribing accurate values to the space-time co-ordinates, as well as to the momentum-energy components before and after the process. The reciprocal uncertainty which always affects the values of these quantities is, as will be clear from the preceding analysis, essentially an outcome of the limited accuracy with which changes in energy and momentum can be defined, when the wave-fields used for the determination of the space-time co-ordinates of the particle are sufficiently small.

In using an optical instrument for determinations of position, it is necessary to remember that the formation of the image always requires a convergent beam of light. Denoting by λ the wave-length of the radiation used, and by ϵ the so-called numerical aperture, that is, the sine of half the angle of convergence, the resolving power of a microscope is given by the well-known expression $\lambda/2\epsilon$. Even if the object is illuminated by parallel light, so that the momentum h/λ of the incident light quantum is known both as regards magnitude and direction, the finite value of the aperture will prevent an exact knowledge of the recoil accompanying the scattering. Also, even if the momentum of the particle were accurately known before the scattering process, our knowledge of the component of momentum parallel to the focal plane after the observation would be affected by an uncertainty amounting to $2\epsilon h/\lambda$. The product of the least inaccuracies with which the positional co-ordinate and the component of momentum in a definite direction can be ascertained is therefore just given by formula (2). One might perhaps expect that in estimating the accuracy of determining the position, not only the convergence but also the length of the wave-train has to be taken into account, because the particle could change its place during the finite time of illumination. Due to the fact, however, that the exact knowledge of the wave-length is immaterial for the above estimate, it will be realized that for any value of the aperture the wave-train can always be taken so short that a change of position of the particle during the time of observation may be neglected in comparison to the lack of sharpness inherent in the determination of position due to the finite resolving power of the microscope.

In measuring momentum with the aid of the Doppler effect—with due regard to the Compton effect—one will employ a parallel wave-train. For the accuracy, however, with which the change in wave-length of the scattered radiation can be measured the extent of the wave-train in the direction of propagation is essential. If we assume that the directions of the incident and scattered radiation are parallel and opposite, respectively, to the direction of the position co-ordinate and momentum component to be measured, then $c\lambda/2l$ can be taken as a measure of the accuracy in the determination of the velocity, where l

denotes the length of the wave-train. For simplicity, we here have regarded the velocity of light as large compared to the velocity of the particle. If m represents the mass of the particle, then the uncertainty attached to the value of the momentum after observation is $cm\lambda/2l$. In this case the magnitude of the recoil, $2h/\lambda$, is sufficiently well defined in order not to give rise to an appreciable uncertainty in the value of the momentum of the particle after observation. Indeed, the general theory of the Compton effect allows us to compute the momentum components in the direction of the radiation before and after the recoil from the wave-lengths of the incident and scattered radiation. Even if the positional co-ordinates of the particle were accurately known in the beginning, our knowledge of the position after observation nevertheless will be affected by an uncertainty. Indeed, on account of the impossibility of attributing a definite instant to the recoil, we know the mean velocity in the direction of observation during the scattering process only with an accuracy $2h/m\lambda$. The uncertainty in the position after observation hence is $2hl/mc\lambda$. Here, too, the product of the inaccuracies in the measurement of position and momentum is thus given by the general formula (2).

Just as in the case of the determination of position, the time of the process of observation for the determination of momentum may be made as short as is desired, if only the wave-length of the radiation used is sufficiently small. The fact that the recoil then gets larger does not, as we have seen, affect the accuracy of measurement. It should further be mentioned, that in referring to the velocity of a particle as we have here done repeatedly, the purpose has only been to obtain a connection with the ordinary space-time description convenient in this case. As it appears already from the considerations of de Broglie mentioned above, the concept of velocity must always in the quantum theory be handled with caution. It will also be seen that an unambiguous definition of this concept is excluded by the quantum postulate. This is particularly to be remembered when comparing the results of successive observations. Indeed, the position of an individual at two given moments can be measured with any desired degree of accuracy; but if, from such measurements, we would calculate the velocity of the individual in the ordinary way, it must be clearly realized that we are dealing with an abstraction, from which no unambiguous information concerning the previous or future behaviour of the individual can be obtained.

According to the above considerations regarding the possibilities of definition of the properties of individuals, it will obviously make no difference in the discussion of the accuracy of measurements of position and momentum of a particle if collisions with other material particles are considered instead of scattering of radiation. In both cases, we see that the uncertainty in question equally affects the description of the agency of measurement and of the object. In fact, this uncertainty cannot be avoided in a description of the behaviour of individuals with respect to a co-ordinate system fixed in the ordinary way by means of solid bodies and unperturbable clocks. The experimental devices—opening and closing of apertures, etc.—are seen to permit only conclusions regarding the space-time extension of the associated wave-fields.

In tracing observations back to our sensations, once more regard has to be taken to the quantum postulate in connection with the perception of the agency of observation, be it through its direct action upon the eye or by means of suitable auxiliaries such as photographic plates, Wilson clouds, etc. It is easily seen, however, that the resulting additional statistical element will not influence the uncertainty in the description of the object. It might even be conjectured that the arbitrariness in what is regarded as object and what as agency of observation would open up a possibility of avoiding this uncertainty altogether. In connection with the measurement of the position of a particle, one might, for example, ask whether the momentum transmitted by the scattering could not be determined by means of the conservation theorem from a measurement of the change of momentum of the microscope—including light source and photographic plate—during the process of observation. A closer investigation shows, however, that such a measurement is impossible, if at the same time one wants to know the position of the microscope with sufficient accuracy. In fact, it follows from the experiences which have found expression in the wave theory of matter that the position of the centre of gravity of a body and its total momentum can only be defined within the limits of reciprocal accuracy given by relation (2).

Strictly speaking, the idea of observation belongs to the causal space-time way of description. Due to the general character of relation (2), however, this idea can be consistently utilized also in the quantum theory, if only the uncertainty expressed through this relation is taken into account. As remarked by Heisenberg, one may even obtain an instructive illustration of the quantum-theoretical description of atomic (microscopic) phenomena by comparing this uncertainty with the uncertainty, due to imperfect measurements, inherently contained in any observation as considered in the ordinary description of natural phenomena. He remarks on that occasion that even in the case of macroscopic phenomena we may say, in a certain sense, that they are created by repeated observations. It must not be forgotten, however, that in the classical theories any succeeding observation permits a prediction of future events with ever-increasing accuracy, because it improves our knowledge of the initial state of the system. According to the quantum theory, just the impossibility of neglecting the interaction with the agency of measurement means that every observation introduces a new uncontrollable element. Indeed, it follows from the above considerations that the measurement of the positional co-ordinates of a particle is accompanied not only by a finite change in the dynamical variables, but also the fixation of its position means a complete rupture in the causal description of its dynamical behaviour, while the determination of its momentum always implies a gap in the knowledge of its spatial propagation. Just this situation brings out most strikingly the complementary character of the description of atomic phenomena which appears as an inevitable consequence of the contrast between the quantum postulate and the distinction between object and agency of measurement, inherent in our very idea of observation.

4. CORRESPONDENCE PRINCIPLE AND MATRIX THEORY

Hitherto we have only regarded certain general features of the quantum problem. The situation implies, however, that the main stress has to be laid on the formulation of the laws governing the interaction between the objects which we symbolize by the abstractions of isolated particles and radiation. Points of attack for this formulation are presented in the first place by the problem of atomic constitution. As is well known, it has been possible here, by means of an elementary use of classical concepts and in harmony with the quantum postulate, to throw light on essential aspects of experience. For example, the experiments regarding the excitation of spectra by electronic impacts and by radiation are adequately accounted for on the assumption of discrete stationary states and individual transition processes. This is primarily due to the circumstance that in these questions no closer description of the space-time behaviour of the processes is required.

Here the contrast with the ordinary way of description appears strikingly in the circumstance that spectral lines, which on the classical view would be ascribed to the same state of the atom, will, according to the quantum postulate, correspond to separate transition processes, between which the excited atom has a choice. Notwithstanding this contrast, however, a formal connection with the classical ideas could be obtained in the limit where the relative difference in the properties of neighbouring stationary states vanishes asymptotically and where in statistical applications the discontinuities may be disregarded. Through this connection it was possible to a large extent to interpret the regularities of spectra on the basis of our ideas about the structure of the atom.

The aim of regarding the quantum theory as a rational generalization of the classical theories led to the formulation of the so-called correspondence principle. The utilization of this principle for the interpretation of spectroscopic results was based on a symbolical application of classical electrodynamics, in which the individual transition processes were each associated with a harmonic in the motion of the atomic particles to be expected according to ordinary mechanics. Except in the limit mentioned, where the relative difference between adjacent stationary states may be neglected, such a fragmentary application of the classical theories could only in certain cases lead to a strictly quantitative description of the phenomena. Especially the connection developed by Ladenburg and Kramers between the classical treatment of dispersion and the statistical laws governing the radiative transition processes formulated by Einstein should be mentioned here. Although it was just Kramers' treatment of dispersion that gave important hints for the rational development of correspondence considerations, it is only through the quantum-theoretical methods created in the last few years that the general aims laid down in the principle mentioned have obtained an adequate formulation.

As is known, the new development was commenced in a fundamental paper by Heisenberg, where he succeeded in emancipating himself completely from the classical concept of motion by replacing from the very start the ordinary kinematical and mechanical quantities by symbols which refer directly to the individual processes demanded by the quantum postulate. This was accomplished by substituting for the Fourier development of a classical mechanical quantity a matrix scheme, the elements of which symbolize purely harmonic vibrations and are associated with the possible transitions between stationary states. By requiring that the frequencies ascribed to the elements must always obey the combination principle for spectral lines, Heisenberg could introduce simple rules of calculation for the symbols which permit a direct quantum-theoretical transcription of the fundamental equations of classical mechanics. This ingenious attack on the dynamical problem of atomic theory proved itself from the beginning to be an exceedingly powerful and fertile method for interpreting quantitatively the experimental results. Through the work of Born and Jordan, as well as of Dirac, the theory was given a formulation which can compete with classical mechanics as regards generality and consistency. Especially, the element characteristic of the quantum theory, Planck's constant, appears explicitly only in the algorithms to which the symbols, the so-called matrices, are subjected. In fact, matrices, which represent canonically conjugated variables in the sense of the Hamiltonian equations, do not obey the commutative law of multiplication, but two such quantities, q and p, have to fulfil the exchange rule

$$pq-qp=\sqrt{-1}\,\frac{h}{2\pi}. \qquad \ldots\ldots(3)$$

Indeed, this exchange relation expresses strikingly the symbolical character of the matrix formulation of the quantum theory. The matrix theory has often been called a calculus with directly observable quantities. It must be remembered, however, that the procedure described is limited just to those problems, in which in applying the quantum postulate the space-time description may largely be disregarded, and the question of observation in the proper sense therefore placed in the background.

In pursuing further the correspondence of the quantum laws with classical mechanics, the stress placed on the statistical character of the quantum-theoretical description, which is brought in by the quantum postulate, has been of fundamental importance. Here the generalization of the symbolical method made by Dirac and Jordan represented a great progress by making possible the operation with matrices, which are not arranged according to the stationary states, but where the possible values of any set of variables may appear as indices of the matrix elements. In analogy to the interpretation considered in the original form of the theory of the "diagonal elements" connected only with a single stationary state, as time averages of the quantity to be represented, the general transformation theory of matrices permits the representation of such averages of a mechanical quantity, in the calculation of which any set of variables characterizing the "state" of the system has given values, while the canonically conjugated variables are allowed to take all possible values. On the basis of the procedure developed by these authors and in close connection with ideas of Born and Pauli, Heisenberg has in the paper already cited above attempted a closer analysis of the physical content of the quantum theory, especially in view of the apparently paradoxical character of the exchange relation (3). In this connection he has formulated the relation

$$\Delta q \Delta p \sim h \qquad \text{......(4)}$$

as the general expression for the maximum accuracy with which two canonically conjugated variables can simultaneously be observed. In this way Heisenberg has been able to elucidate many paradoxes appearing in the application of the quantum postulate, and to a large extent to demonstrate the consistency of the symbolic method. In connection with the complementary nature of the quantum-theoretical description, we must, as already mentioned, constantly keep the possibilities of definition as well as of observation before the mind. For the discussion of just this question the method of wave mechanics developed by Schrödinger has, as we shall see, proved of great help. It permits a general application of the principle of superposition also in the problem of interaction, thus offering an immediate connection with the above considerations concerning radiation and free particles. Below we shall return to the relation of wave mechanics to the general formulation of the quantum laws by means of the transformation theory of matrices.

5. WAVE MECHANICS AND THE QUANTUM POSTULATE

Already in his first considerations concerning the wave theory of material particles, de Broglie pointed out that the stationary states of an atom may be visualized as an interference effect of the phase wave associated with a bound electron. It is true that this point of view at first did not, as regards quantitative results, lead beyond the earlier methods of quantum theory, to the development of which Sommerfeld has contributed so essentially. Schrödinger, however, succeeded in developing a wave-theoretical method which has opened up new aspects, and has proved to be of decisive importance for the great progress in atomic physics during the last years. Indeed, the proper vibrations of the Schrödinger wave equation have been found to furnish a representation of the stationary states of an atom meeting all requirements. The energy of each state is connected with the corresponding period of vibration according to the general quantum relation (1). Furthermore, the number of nodes in the various characteristic vibrations gives a simple interpretation to the concept of quantum number which was already known from the older methods, but at first did not seem to appear in the matrix formulation. In addition, Schrödinger could associate with the solutions of the wave equation a continuous distribution of charge and current which, if applied to a characteristic vibration, represents the electrostatic and magnetic properties of an atom in the corresponding stationary state. Similarly, the superposition of two characteristic solutions corresponds to a continuous vibrating distribution of electrical charge, which on classical electrodynamics would give rise to an emission of radiation, illustrating instructively the consequences of the quantum postulate and the correspondence requirement regarding the transition process between two stationary states formulated in matrix mechanics. Another application of the method of Schrödinger, important for the further development, has been made by Born in his investigation of the problem of collisions between atoms and free electric particles. In this connection he succeeded in obtaining a statistical interpretation of the wave functions, allowing a calculation of the probability of the individual transition processes required by the quantum postulate. This includes a wave-mechanical formulation of the adiabatic principle of Ehrenfest, the fertility of which appears strikingly in the promising investigations of Hund on the problem of the formation of molecules.

In view of these results, Schrödinger has expressed the hope that the development of the wave theory will eventually remove the irrational element expressed by the quantum postulate and open the way for a complete description of atomic phenomena along the line of the classical theories. In support of this view, Schrödinger, in a recent paper, emphasizes the fact that the discontinuous exchange of energy between atoms required by the quantum postulate, from the point of view of the wave theory, is replaced by a simple resonance phenomenon. In particular, the idea of individual stationary states would be an illusion and its applicability only an illustration of the resonance mentioned. It must be kept in mind, however, that just in the resonance problem mentioned we are concerned with a closed system which, according to the view presented here, is not accessible to observation. In fact, wave mechanics, just as the matrix theory, on this view represents a symbolic transcription of the problem of motion of classical mechanics adapted to the requirements of quantum theory and only to be interpreted by an explicit use of the quantum postulate. Indeed, the two formulations of the interaction problem might be said to be complementary in the same sense as the wave and particle idea in the description of the free individuals. The apparent contrast in the utilization of the energy concept in the two theories is just connected with this difference in the starting-point.

The fundamental difficulties opposing a space-time description of a system of particles in interaction appear at once from the inevitability of the superposition principle in the description of the behaviour of individual particles. Already for a free particle the knowledge of energy and momentum excludes, as we have seen, the exact knowledge of its space-time co-ordinates. This implies that an immediate utilization of the concept of energy in connection with the classical idea of the potential energy of the system is excluded. In the Schrödinger wave equation these difficulties are avoided by replacing the classical expression of the Hamiltonian by a differential operator by means of the relation

$$p=\sqrt{-1}\,\frac{h}{2\pi}\,\frac{\delta}{\delta q}, \qquad \ldots\ldots(5)$$

where p denotes a generalized component of momentum and q the canonically conjugated variable. Here the negative value of the energy is regarded as conjugated to the time. So far, in the wave equation, time and space as well as energy and momentum are utilized in a purely formal way.

The symbolical character of Schrödinger's method appears not only from the circumstance that its simplicity, similarly to that of the matrix theory, depends essentially upon the use of imaginary arithmetic quantities. But above all there can be no question of an immediate connection with our ordinary conceptions because the "geometrical" problem represented by the wave equation is associated with the so-called co-ordinate space, the number of dimensions of which is equal to the number of degrees of freedom of the system, and, hence, in general greater than the number of dimensions of ordinary space. Further, Schrödinger's formulation of the interaction problem, just as the formulation offered by matrix theory, involves a neglect of the finite velocity of propagation of the forces claimed by relativity theory.

On the whole, it would scarcely seem justifiable, in the case of the interaction problem, to demand a visualization by means of ordinary space-time pictures. In fact, all our knowledge concerning the internal properties of atoms is derived from experiments on their radiation or collision reactions, such that the interpretation of experimental facts ultimately depends on the abstractions of radiation in free space, and free material particles. Hence, our whole space-time view of physical phenomena, as well as the definition of energy and momentum, depends ultimately upon these abstractions. In judging the applications of these auxiliary ideas, we should only demand inner consistency, in which connection special regard has to be paid to the possibilities of definition and observation.

In the characteristic vibrations of Schrödinger's wave equation we have, as mentioned, an adequate representation of the stationary states of an atom allowing an unambiguous definition of the energy of the system by means of the general quantum relation (1). This entails, however, that in the interpretation of observations a fundamental renunciation regarding the space-time description is unavoidable. In fact, the consistent application of the concept of stationary states excludes, as we shall see, any specification regarding the behaviour of the separate particles in the atom. In problems where a description of this behaviour is essential, we are bound to use the general solution of the wave equation which is obtained by superposition of characteristic solutions. We meet here with a complementarity of the possibilities of definition quite analogous to that which we have considered earlier in connection with the properties of light and free material particles. Thus, while the definition of energy and momentum of individuals is attached to the idea of a harmonic elementary wave, every space-time feature of the description of phenomena is, as we have seen, based on a consideration of the interferences taking place inside a group of such elementary waves. Also in the present case the agreement between the possibilities of observation and those of definition can be directly shown.

According to the quantum postulate any observation regarding the behaviour of the electron in the atom will be accompanied by a change in the state of the atom. As stressed by Heisenberg, this change will, in the case of atoms in stationary states of low quantum number, consist in general in the ejection of the electron from the atom. A description of the "orbit" of the electron in the atom with the aid of subsequent observations is, hence, impossible in such a case. This is connected with the circumstance that from characteristic vibrations with only a few nodes no wave packages can be built up which would even approximately represent the "motion" of a particle. The complementary nature of the description, however, appears particularly in that the use of observations concerning the behaviour of particles in the atom rests on the possibility of neglecting, during the process of observation, the interaction between the particles, thus regarding them as free. This requires, however, that the duration of the process is short compared with the natural periods of the atom, which again means that the uncertainty in the knowledge of the energy transferred in the process is large compared to the energy differences between neighbouring stationary states.

In judging the possibilities of observation it must, on the whole, be kept in mind that the wave-mechanical solutions can be visualized only in so far as they can be described with the aid of the concept of free particles. Here the difference between classical mechanics and the quantum-theoretical treatment of the problem of interaction appears most strikingly. In the former such a restriction is unnecessary because the "particles" are here endowed with an immediate "reality", independently of their being free or bound. This situation is particularly important in connection with the consistent utilization of Schrödinger's electric density as a measure of the probability for electrons being present within given space regions of the atom. Remembering the restriction mentioned, this interpretation is seen to be a simple consequence of the assumption that the probability of the presence of a free electron is expressed by the electric density associated with the wave-field in a similar way to that by which the probability of the presence of a light quantum is given by the energy density of the radiation.

As already mentioned, the means for a general consistent utilization of the classical concepts in the quantum theory have been created through the transformation theory of Dirac and Jordan, by the aid of which Heisenberg has formulated his general uncertainty relation (4). In this theory also the Schrödinger wave equation has obtained an instructive application. In fact, the characteristic solutions of this equation appear as auxiliary functions which define a transformation from matrices with indices representing the energy values of the system to other matrices, the indices of which are the possible values of the space co-ordinates. It is also of interest in this connection to mention that Jordan and Klein have recently arrived at the formulation of the problem of interaction expressed by the Schrödinger wave equation, taking as starting-point the wave representation of individual particles and applying a symbolic method closely related to the deep-going treatment of the radiation problem developed by Dirac from the point of view of the matrix theory, to which we shall return below.

6. REALITY OF STATIONARY STATES

In the conception of stationary states we are, as mentioned, concerned with a characteristic application of the quantum postulate. By its very nature this conception means a complete renunciation as regards a time description. From the point of view taken here, just this renunciation forms the necessary condition for an unambiguous definition of the energy of the atom. Moreover, the conception of a stationary state involves, strictly speaking, the exclusion of all interactions with individuals not belonging to the system. The fact that such a closed system is associated with a particular energy value may be considered as an immediate expression for the claim of causality contained in the theorem of conservation of energy. This circumstance justifies the assumption of the supra-mechanical stability of the stationary states, according to which the atom, before as well as after an external influence, always will be found in a well-defined state, and which forms the basis for the use of the quantum postulate in problems concerning atomic structure.

In a judgment of the well-known paradoxes which this assumption entails for the description of collision and radiation reactions, it is essential to consider the limitations of the possibilities of definition of the reacting free individuals, which is expressed by relation (2). In fact, if the definition of the energy of the reacting individuals is to be accurate to such a degree as to entitle us to speak of conservation of energy during the reaction, it is necessary, according to this relation, to co-ordinate to the reaction a time interval long compared to the vibration period associated with the transition process, and connected with the energy difference between the stationary states according to relation (1). This is particularly to be remembered when considering the passage of swiftly moving particles through an atom. According to the ordinary kinematics, the effective duration of such a passage would be very small as compared with the natural periods of the atom, and it seemed impossible to reconcile the principle of conservation of energy with the assumption of the stability of stationary states. In the wave representation, however, the time of reaction is immediately connected with the accuracy of the knowledge of the energy of the colliding particle, and hence there can never be the possibility of a contradiction with the law of conservation. In connection with the discussion of paradoxes of the kind mentioned, Campbell suggested the view that the conception of time itself may be essentially statistical in nature. From the view advanced here, according to which the foundation of space-time description is offered by the abstraction of free individuals, a fundamental distinction between time and space, however, would seem to be excluded by the relativity requirement. The singular position of the time in problems concerned with stationary states is, as we have seen, due to the special nature of such problems.

The application of the conception of stationary states demands that in any observation, say by means of collision or radiation reactions, permitting a distinction between different stationary states, we are entitled to disregard the previous history of the atom. The fact that the symbolical quantum theory methods ascribe a particular phase to each stationary state the value of which depends upon the previous history of the atom, would for the first moment seem to contradict the very idea of stationary states. As soon as we are really concerned with a time problem, however, the consideration of a strictly closed system is excluded. The use of simply harmonic proper vibrations in the interpretation of observations means, therefore, only a suitable idealization which in a more rigorous discussion must always be replaced by a group of harmonic vibrations, distributed over a finite frequency interval. Now, as already mentioned, it is a general consequence of the superposition principle that it has no sense to co-ordinate a phase value to the group as a whole, in the same manner as may be done for each elementary wave constituting the group.

This inobservability of the phase, well known from the theory of optical instruments, is brought out in a particularly simple manner in a discussion of the Stern-Gerlach experiment, so important for the investigation of the properties of single atoms. As pointed out by Heisenberg, atoms with different orientation in the field may only be separated if the deviation of the beam is larger than the diffraction at the slit of the de Broglie waves representing the translational motion of the atoms. This condition means, as a simple calculation shows, that the product of the time of passage of the atom through the field, and the uncertainty due to the finite width of the beam of its energy in the field, is at least equal to the quantum of action. This result was considered by Heisenberg as a support of relation (2) as regards the reciprocal uncertainties of energy and time values. It would seem, however, that here we are not simply dealing with a measurement of the energy of the atom at a given time. But since the period of the proper vibrations of the atom in the field is connected with the total energy by relation (1), we realize that the condition for separability mentioned just means the loss of the phase. This circumstance removes also the apparent contradictions, arising in certain problems concerning the coherence of resonance radiation, which have been discussed frequently, and were also considered by Heisenberg.

To consider an atom as a closed system, as we have done above, means to neglect the spontaneous emission of radiation which even in the absence of external influences puts an upper limit to the lifetime of the stationary states. The fact that this neglect is justified in many applications is connected with the circumstance that the coupling between the atom and the radiation field, which is to be expected on classical electrodynamics, is in general very small compared to the coupling between the particles in the atom. It is, in fact, possible in a description of the state of an atom to a considerable extent to neglect the reaction of radiation, thus disregarding the unsharpness in the energy values connected with the lifetime of the stationary states according to relation (2). This is the reason why it is possible to draw conclusions concerning the properties of radiation by using classical electrodynamics.

The treatment of the radiation problem by the new quantum-theoretical methods meant, to begin with, just a quantitative formulation of this correspondence consideration. This was the very starting-point of the original considerations of Heisenberg. It may also be mentioned that an instructive analysis of Schrödinger's treatment of the radiation phenomena from the point of view of the correspondence principle has been recently given by Klein. In the more rigorous form of the theory developed by Dirac, the radiation field itself is included in the closed system under consideration. Thus it became possible in a rational way to take account of the individual character of radiation demanded by the quantum theory and to build up a dispersion theory, in which the finite width of the spectral lines is taken into consideration. The renunciation regarding space-time pictures characterizing this treatment would seem to offer a striking indication of the complementary character of the quantum theory. This is particularly to be borne in mind in judging the radical departure from the causal description of Nature met with in radiation phenomena, to which we have referred above in connection with the excitation of spectra.

In view of the asymptotic connection of atomic properties with classical electrodynamics, demanded by the correspondence principle, the reciprocal exclusion of the conception of stationary states and the description of the behaviour of individual particles in the atom might be regarded as a difficulty. In fact, the connection in question means that in the limit of large quantum numbers where the relative difference between adjacent stationary states vanishes asymptotically, mechanical pictures of electronic motion may be rationally utilized. It must be emphasized, however, that this connection cannot be regarded as a gradual transition towards classical theory in the sense that the quantum postulate would lose its significance for high quantum numbers. On the contrary, the conclusions obtained from the correspondence principle with the aid of classical pictures depend just upon the assumptions that the conception of stationary states and of individual transition processes are maintained even in this limit.

This question offers a particularly instructive example for the application of the new methods. As shown by Schrödinger, it is possible, in the limit mentioned, by superposition of proper vibrations to construct wave-groups small in comparison to the "size" of the atom, the propagation of which indefinitely approaches the classical picture of moving material particles, if the quantum numbers are chosen sufficiently large. In the special case of a simple harmonic vibrator, he was able to show that such wave-groups will keep together even for any length of time, and will oscillate to and fro in a manner corresponding to the classical picture of the motion. This circumstance Schrödinger has regarded as a support of his hope of constructing a pure wave theory without referring to the quantum postulate. As emphasized by Heisenberg, the simplicity of the case of the oscillator, however, is exceptional and intimately connected with the harmonic nature of the corresponding classical motion. Nor is there in this example any possibility for an asymptotical approach towards the problem of free particles. In general, the wave-group will gradually spread over the whole region of the atom, and the "motion" of a bound electron can only be followed during a number of periods, which is of the order of magnitude of the quantum numbers associated with the proper vibrations. This question has been more closely investigated in a recent paper by Darwin which contains a number of instructive examples of the behaviour of wave groups. From the viewpoint of the matrix theory a treatment of analogous problems has been carried out by Kennard.

Here again we meet with the contrast between the wave-theory superposition principle and the assumption of the individuality of particles with which we have been concerned already in the case of free particles. At the same time the asymptotical connection with the classical theory, to which a distinction between free and bound particles is unknown, offers the possibility of a particularly simple illustration of the above considerations regarding the consistent utilization of the concept of stationary states. As we have seen, the identification of a stationary state by means of collision or radiation reactions implies a gap in the time description, which is at least of the order of magnitude of the periods associated with transitions between stationary states. Now, in the limit of high quantum numbers these periods may be interpreted as periods of revolution. Thus we see at once that no causal connection can be obtained between observations leading to the fixation of a stationary state and earlier observations on the behaviour of the separate particles in the atom.

Summarizing, it might be said that the concepts of stationary states and individual transition processes within their proper field of application possess just as much or as little "reality" as the very idea of individual particles. In both cases we are concerned with a demand of causality complementary to the space-time description, the adequate application of which is limited only by the restricted possibilities of definition and of observation.

7. THE PROBLEM OF THE ELEMENTARY PARTICLES

When due regard is taken of the complementary feature required by the quantum postulate, it seems, in fact, possible with the aid of the symbolic methods to build up a consistent theory of atomic phenomena, which may be considered as a rational generalization of the causal space-time description of classical physics. This view does not mean, however, that classical electron theory may be regarded simply as the limiting case of a vanishing quantum of action. Indeed, the connection of the latter theory with experience is based on assumptions which can scarcely be separated from the group of problems of the quantum theory. A hint in this direction was already given by the well-known difficulties met with in the attempts to account for the individuality of ultimate electrical particles on general mechanical and electrodynamical principles. In this respect also, the general relativity theory of gravitation has not fulfilled expectations. A satisfactory solution of the problems touched upon would seem to be possible only by means of a rational quantum-theoretical transcription of the general field theory, in which the ultimate quantum of electricity has found its natural position as an expression of the feature of individuality characterizing the quantum theory. Recently Klein has directed attention to the possibility of connecting this problem with the five-dimensional unified representation of electromagnetism and gravitation proposed by Kaluza. In fact, the conservation of electricity appears in this theory as an analogue to the conservation theorems for energy and momentum. Just as these concepts are complementary to the space-time description, the appropriateness of the ordinary four-dimensional description as well as its symbolical utilization in the quantum theory would, as Klein emphasizes, seem to depend essentially on the circumstance that in this description electricity always appears in well-defined units, the conjugated fifth dimension being as a consequence not open to observation.

Quite apart from these unsolved deep-going problems, the classical electron theory up to the present time has been the guide for a further development of the correspondence description in connection with the idea first advanced by Compton that the ultimate electrical particles, besides their mass and charge, are endowed with a magnetic moment due to an angular momentum determined by the quantum of action. This assumption, introduced with striking success by Goudsmit and

Uhlenbeck into the discussion of the origin of the anomalous Zeeman effect, has proved most fruitful in connection with the new methods, as shown especially by Heisenberg and Jordan. One might say, indeed, that the hypothesis of the magnetic electron, together with the resonance problem elucidated by Heisenberg, which occurs in the quantum-theoretical description of the behaviour of atoms with several electrons, have brought the correspondence interpretation of the spectral laws and the periodic system to a certain degree of completion. The principles underlying this attack have even made it possible to draw conclusions regarding the properties of atomic nuclei. Thus Dennison, in connection with ideas of Heisenberg and Hund, has succeeded recently in a very interesting way in showing how the explanation of the specific heat of hydrogen, hitherto beset with difficulties, can be harmonized with the assumption that the proton is endowed with a moment of momentum of the same magnitude as that of the electron. Due to its larger mass, however, a magnetic moment much smaller than that of the electron must be associated with the proton.

The insufficiency of the methods hitherto developed as concerns the problem of the elementary particles appears in the questions just mentioned from the fact that they do not allow of an unambiguous explanation of the difference in the behaviour of the electric elementary particles and the "individuals" symbolized through the conception of light quanta expressed in the so-called exclusion principle formulated by Pauli. In fact, we meet in this principle, so important for the problem of atomic structure as well as for the recent development of statistical theories, with one among several possibilities, each of which fulfils the correspondence requirement. Moreover, the difficulty of satisfying the relativity requirement in quantum theory appears in a particularly striking light in connection with the problem of the magnetic electron. Indeed, it seemed not possible to bring the promising attempts made by Darwin and Pauli in generalizing the new methods to cover this problem naturally, in connection with the relativity kinematical consideration of Thomas so fundamental for the interpretation of experimental results. Quite recently, however, Dirac has been able successfully to attack the problem of the magnetic electron through a new ingenious extension of the symbolical method and so to satisfy the relativity requirement without abandoning the agreement with spectral evidence. In this attack not only the imaginary complex quantities appearing in the earlier procedures are involved, but his fundamental equations themselves contain quantities of a still higher degree of complexity that are represented by matrices.

Already the formulation of the relativity argument implies essentially the union of the space-time co-ordination and the demand of causality characterizing the classical theories. In the adaptation of the relativity requirement to the quantum postulate, we must therefore be prepared to meet with a renunciation as to visualization in the ordinary sense going still further than in the formulation of the quantum laws considered here. Indeed, we find ourselves here on the very path taken by Einstein of adapting our modes of perception borrowed from the sensations to the gradually deepening knowledge of the laws of Nature. The hindrances met with on this path originate above all in the fact that, so to say, every word in the language refers to our ordinary perception. In the quantum theory we meet this difficulty at once in the question of the inevitability of the feature of irrationality characterizing the quantum postulate. I hope, however, that the idea of complementarity is suited to characterize the situation, which bears a deep-going analogy to the general difficulty in the formation of human ideas, inherent in the distinction between subject and object.

III

The Quantum of Action and the Description of Nature

(1929)

In the history of science there are few events which, in the brief span of a generation, have had such extraordinary consequences as Planck's discovery of the elementary quantum of action. Not only does this discovery, to an ever increasing degree, form the background for the ordering of our experience concerning atomic phenomena, the knowledge of which has been so amazingly extended in the last thirty years, but, at the same time, it has brought about a complete revision of the foundations underlying our description of natural phenomena. We are dealing here with an unbroken development of points of view and conceptual aids which, beginning with the fundamental works of Planck on black body radiation, has reached a temporary climax, in recent years, in the formulation of a symbolic quantum mechanics. This theory may be regarded as a natural generalization of the classical mechanics with which in beauty and self-consistency it may well be compared.

This goal has not been attained, still, without a renunciation of the causal space-time mode of description that characterizes the classical physical theories which have experienced such a profound clarification through the theory of relativity. In this respect, the quantum theory may be said to be a disappointment, for the atomic theory arose just from the attempt to accomplish such a description also in the case of phenomena which, in our immediate sense impressions, do not appear as motions of material bodies. From the very beginning, however, one was not unprepared in this domain to come upon a failure of the forms of perception adapted to our ordinary sense impressions. We know now, it is true, that the often expressed scepticism with regard to the reality of atoms was exaggerated; for, indeed, the wonderful development of the art of experimentation has enabled us to study the effects of individual atoms. Nevertheless, the very recognition of the limited divisibility of physical processes, symbolized by the quantum of action, has justified the old doubt as to the range of our ordinary forms of perception when applied to atomic phenomena. Since, in the observation of these phenomena, we cannot neglect the interaction between the object and the instrument of observation, the question of the possibilities of observation again comes to the foreground. Thus, we meet here, in a new light, the problem of the objectivity of phenomena which has always attracted so much attention in philosophical discussion.

This being the state of affairs, it is not surprising that, in all rational applications of the quantum theory, we have been concerned with essentially statistical problems. Indeed, in the original researches of Planck, it was, above all, the necessity for modifying the classical statistical mechanics which gave rise to the introduction of the quantum of action. This feature, which is characteristic of the quantum theory, is strikingly expressed in the recently renewed discussion on the nature of light and of the elementary particles of matter. Although these questions had apparently found their final solution within the compass of the classical theories, we know now that for material particles as well as for light different conceptual pictures are necessary to account completely for the phenomena and to furnish a unique formulation of the statistical laws which govern the data of observation. The more clearly it appears that a uniform formulation of the quantum theory in classical terms is impossible, the more we admire Planck's happy intuition in coining the term "quantum of action" which directly indicates a renunciation of the action principle, the central position of which in the classical description of nature he himself has emphasized on more than one occasion. This principle symbolizes, as it were, the peculiar reciprocal symmetry relation between the space-time description and the laws of the conservation of energy and momentum, the great fruitfulness of which, already in classical physics, depends upon the fact that one may extensively apply them without following the course of the phenomena in space and time. It is this very reciprocity which has been made use of in a most pregnant way in the quantum-mechanical formalism. As a matter of fact, the quantum of action appears here only in relations in which space-time co-ordinates and momentum-energy components, which are canonically conjugate quantities in the Hamiltonian sense, enter in a symmetrical and reciprocal manner. In addition, the analogy between optics and mechanics, which has proved to be so fruitful for the recent development of the quantum theory, depends intimately upon this reciprocity.

It lies in the nature of physical observation, nevertheless, that all experience must ultimately be expressed in terms of classical concepts, neglecting the quantum of action. It is, therefore, an inevitable consequence of the limited applicability of the classical concepts that the results attainable by any measurement of atomic quantities are subject to an inherent limitation. A profound clarification of this question was recently accomplished with the help of the general quantum-mechanical law, formulated by Heisenberg, according to which the product of the mean errors with which two canonically conjugate mechanical quantities may be simultaneously measured can never be smaller than the quantum of action. Heisenberg has rightly compared the significance of this law of reciprocal uncertainty for estimating the self-consistency of quantum mechanics with the significance of the impossibility of transmitting signals with a velocity greater than that of light for testing the self-consistency of the theory of relativity. In considering the well-known paradoxes which are encountered in the application of the quantum theory to atomic structure, it is essential to remember, in this connection, that the properties of atoms are always obtained by observing their reactions under collisions or under the influence of radiation, and that the above-mentioned limitation on the possibilities of measurement is directly related to the apparent contradictions which have been revealed in the discussion of the nature of light and of material particles. In order to emphasize that we are not concerned here with real contradictions, the author suggested in an earlier article the term "complementarity". In consideration of the above-mentioned reciprocal symmetry which occurs already in classical mechanics, perhaps the term "reciprocity" is more suitable for expressing the state of affairs with which we are dealing. At the conclusion of the paper referred to, it was pointed out that a close connection exists between the failure of our forms of perception, which is founded on the impossibility of a strict separation of phenomena and means of observation, and the general limits of man's capacity to create concepts, which have their roots in our differentiation between subject and object. Indeed, the epistemological and psychological questions which arise here lie perhaps outside the range of physics proper. Yet, on this special occasion, I should like to be permitted to go somewhat more deeply into these ideas.

The epistemological problem under discussion may be characterized briefly as follows: For describing our mental activity, we require, on one hand, an objectively given content to be placed in opposition to a perceiving subject, while, on the other hand, as is already implied in such an assertion, no sharp separation between object and subject can be maintained, since the perceiving subject also belongs to our mental content. From these circumstances follows not only the relative meaning of every concept, or rather of every word, the meaning depending upon our arbitrary choice of view point, but also that we must, in general, be prepared to accept the fact that a complete elucidation of one and the same object may require diverse points of view which defy a unique description. Indeed, strictly speaking, the conscious analysis of any concept stands in a relation of exclusion to its immediate application. The necessity of taking recourse to a complementary, or reciprocal, mode of description is perhaps most familiar to us from psychological problems. In opposition to this, the feature which characterizes the so-called exact sciences is, in general, the attempt to attain to uniqueness by avoiding all reference to the perceiving subject. This endeavour is found most consciously, perhaps, in the mathematical symbolism which sets up for our contemplation an ideal of objectivity to the attainment of which scarcely any limits are set, so long as we remain within a self-contained field of applied logic. In the natural sciences proper, however, there can be no question of a strictly self-contained field of application of the logical principles, since we must continually count on the appearance of new facts, the inclusion of which within the compass of our earlier experience may require a revision of our fundamental concepts.

We have recently experienced such a revision in the rise of the theory of relativity which, by a profound analysis of the problem of observation, was destined to reveal the subjective character of all the concepts of classical physics. In spite of the great demands that it makes upon our power of abstraction, the theory of relativity approaches, in a particularly high degree, the classical ideal of unity and causality in the description of nature. Above all, the conception of the objective reality of the phenomena open to observation is still rigidly maintained. As Einstein has emphasized, the assumption that any observation ultimately depends upon the coincidence in space and time of the object and the means of observation and that, therefore, any observation is definable independently of the reference system of the observer, is, indeed, fundamental for the whole theory of relativity. However, since the discovery of the quantum of action, we know that the classical ideal cannot be attained in the description of atomic phenomena.

In particular, any attempt at an ordering in space-time leads to a break in the causal chain, since such an attempt is bound up with an essential exchange of momentum and energy between the individuals and the measuring rods and clocks used for observation; and just this exchange cannot be taken into account if the measuring instruments are to fulfil their purpose. Conversely, any conclusion, based in an unambiguous manner upon the strict conservation of energy and momentum, with regard to the dynamical behaviour of the individual units obviously necessitates a complete renunciation of following their course in space and time. In general, we may say that the suitableness of the causal space-time mode of description for the ordering of our usual experiences depends only upon the smallness of the quantum of action relative to the actions with which we are concerned in ordinary phenomena. Planck's discovery has brought before us a situation similar to that brought about by the discovery of the finite velocity of light. Indeed, the suitability of the sharp distinction between space and time, demanded by our senses, depends entirely upon the smallness of the velocities with which we have to do in daily life compared with the velocity of light. In fact, in the question of the causality of atomic phenomena, the reciprocal character of the results of measurements may no more be neglected than can their relativity in the question of simultaneity.

In considering the resignation with regard to the desires for visualization which give our whole language its character, to which we are compelled by the situation discussed above, it is very instructive that already in simple psychological experiences we come upon fundamental features not only of the relativistic but also of the reciprocal view. The relativity of our perception of motion, with which we become conversant as children when travelling by ship or by train, corresponds to common-place experiences on the reciprocal character of the perception of touch. One need only remember here the sensation, often cited by psychologists, which every one has experienced when attempting to orient himself in a dark room by feeling with a stick. When the stick is held loosely, it appears to the sense of touch to be an object. When, however, it is held firmly, we lose the sensation that it is a foreign body, and the impression of touch becomes immediately localized at the point where the stick is touching the body under investigation. It would scarcely be an exaggeration to maintain, purely from psychological experiences, that the concepts of space and time by their very nature acquire a meaning only because of the possibility of neglecting the interaction with the means of measurement. On the whole, the analysis of our sense impressions discloses a remarkable independence of the psychological foundations of the concepts of space and time, on the one hand, and the conceptions of energy and momentum, based upon actions of force, on the other hand. Above all, however, this domain, as already mentioned, is distinguished by reciprocal relationships which depend upon the unity of our consciousness and which exhibit a striking similarity with the physical consequences of the quantum of action. We are thinking here of well-known characteristics of emotion and volition which are quite incapable of being represented by visualizable pictures. In particular, the apparent contrast between the continuous onward flow of associative thinking and the preservation of the unity of the personality exhibits a suggestive analogy with the relation between the wave description of the motions of material particles, governed by the superposition principle, and their indestructible individuality. The unavoidable influence on atomic phenomena caused by observing them here corresponds to the well-known change of the tinge of the psychological experiences which accompanies any direction of the attention to one of their various elements.

It might still be permitted here briefly to refer to the relation which exists between the regularities in the domain of psychology and the problem of the causality of physical phenomena. When considering the contrast between the feeling of free will, which governs the psychic life, and the apparently uninterrupted causal chain of the accompanying physiological processes, the thought has, indeed, not eluded philosophers that we may be concerned here with an unvisualizable relation of complementarity. Thus, the opinion has often been expressed that a detailed investigation of the processes of the brain, which, although not practicable, is, nevertheless, thinkable, would reveal a causal chain that formed a unique representation of the emotional mental experience. However, such an idealized experiment now appears in a new light, since we have learned, by the discovery of the quantum of action, that a detailed causal tracing of atomic processes is impossible and that any attempt to acquire a knowledge of such processes involves a fundamentally uncontrollable interference with their course. According to the above-mentioned view on the relation between the processes in the brain and the psychical experiences, we must, therefore, be prepared to accept the fact that an attempt to observe the former will bring about an essential alteration in the awareness of volition. Although, in the present case, we can be concerned only with more or less fitting analogies, yet we can hardly escape the conviction that in the facts which are revealed to us by the quantum theory and lie outside the domain of our ordinary forms of perception we have acquired a means of elucidating general philosophical problems.

I hope that the special occasion will excuse a physicist for venturing into a foreign field. Above all, my purpose has been to give expression to our enthusiasm for the prospects which have been opened up for the whole of science. In addition, it has been my desire to emphasize as strongly as possible how profoundly the new knowledge has shaken the foundations underlying the building up of concepts, on which not only the classical description of physics rests but also all our ordinary mode of thinking. It is above all to this emancipation that we owe the wonderful progress in our insight into the phenomena of nature which has been made during the last generation, a progress far exceeding all the hopes which one ventured to cherish just a few years ago. Perhaps the most distinguishing characteristic of the present position of physics is that almost all the ideas which have ever proved to be fruitful in the investigation of nature have found their right place in a common harmony without thereby having diminished their fruitfulness. In gratitude for the possibilities of research which he has opened up before us, his colleagues celebrate to-day the creator of the quantum theory.

IV

The Atomic Theory and the Fundamental Principles underlying the Description of Nature

(1929)

Natural phenomena, as experienced through the medium of our senses, often appear to be extremely variable and unstable. To explain this, it has been assumed, since early times, that the phenomena arise from the combined action and interplay of a large number of minute particles, the so-called atoms, which are themselves unchangeable and stable, but which, owing to their smallness, escape an immediate perception. Quite apart from the fundamental question of whether we are justified in demanding visualizable pictures in fields which lie outside the reach of our senses, the atomic theory originally was of necessity of a hypothetical character; and, since it was believed that a direct insight into the world of atoms would, from the very nature of the matter, never be possible, one had to assume that the atomic theory would always retain this character. However, what has happened in so many other fields has happened also here; because of the development of observational technique, the limit of possible observations has continually been shifted. We need only think of the insight into the structure of the universe which we have gained by the aid of the telescope and the spectroscope, or of the knowledge of the finer structure of organisms which we owe to the microscope. Similarly, the extraordinary development in the methods of experimental physics has made known to us a large number of phenomena which in a direct way inform us of the motions of atoms and of their number. We are aware even of phenomena which with certainty may be assumed to arise from the action of a single atom, or even of a part of an atom. However, at the same time as every doubt regarding the reality of atoms has been removed and as we have gained a detailed knowledge even of the inner structure of atoms, we have been reminded in an instructive manner of the natural limitation of our forms of perception. It is this peculiar situation which I shall attempt to portray here.

Time does not permit of my describing in detail the extraordinary extension of our experience, here dealt with, which is characterized by the discoveries of cathode rays, Röntgen rays, and the radioactive substances. I shall merely remind you of the main features of the picture of the atom which we have gained through these discoveries. Negatively charged particles, the so-called electrons, which are held within the atom by the attraction of a much heavier positively charged atomic nucleus, enter as common building stones in all atoms. The mass of the nucleus determines the atomic weight of the element but has otherwise only a slight influence on the properties of the substance, these depending primarily on the electric charge of the nucleus which, apart from the sign, is always an integral multiple of the charge of an electron. Now, this whole number, which determines how many electrons are present in the neutral atom, has turned out to be just the atomic number that gives the place of the element in the so-called natural system, in which the peculiar relationships of the elements as regards their physical and chemical properties are so appropriately expressed. This interpretation of the atomic number may be said to signify an important step towards the solution of a problem which for a long time has been one of the boldest dreams of natural science, namely, to build up an understanding of the regularities of nature upon the consideration of pure numbers.

The development mentioned above has, to be sure, produced a certain change in the fundamental concepts of the atomic theory. Instead of assuming that the atoms are unchangeable, it is now assumed that it is the parts of the atoms which are constant. In particular, the great stability of the elements depends upon the fact that the atomic nucleus is not affected by the ordinary physical and chemical influences which produce changes only in the binding of the electrons within the atom. While all our experience strengthens the assumption of the permanence of electrons, we know, however, that the stability of atomic nuclei is of a more limited character. Indeed, the peculiar radiations from the radioactive elements provide us with direct evidence of a disruption of atomic nuclei, whereby electrons or positively charged nuclear particles are ejected with great energy. These disintegrations, so far as we are able to judge from all evidence, take place without any external cause. If we have a given number of radium atoms, we can merely say that there is a definite probability that a certain fraction of them will break down during the next second. We shall return later to this peculiar failure of the causal mode of description which we come upon here and which is closely connected with fundamental features of our description of atomic phenomena. Here, I shall call to mind only the important discovery of Rutherford that a disruption of atomic nuclei may, under certain circumstances, be brought about by external influence. As we all know, he succeeded in showing that the nuclei of certain, otherwise stable, elements may be split up when hit by the particles expelled from the radioactive nuclei. This first example of a transmutation of an element, regulated by man, may be said to mark an epoch in the history of natural science and to open up an entirely new field of physics, namely, the exploration of the interior of atomic nuclei. However, I shall not dwell upon the prospects opened up by this new field, but shall confine myself to discussing the general information that we have gained through our endeavours to account for the ordinary physical and chemical properties of the elements on the basis of the conceptions of atomic structure mentioned above.

At first glance, it might appear that the solution of the problem considered would be quite simple. The picture of the atom with which we are dealing is that of a small mechanical system which even resembles in certain main features our own solar system, in the description of which mechanics has won such great triumphs and has given us a principal example of the fulfilment of the claim of causality in ordinary physics. Indeed, from a knowledge of the instantaneous positions and motions of the planets, we can calculate, with apparently unlimited accuracy, their positions and motions at any later time. However, the fact that in such a mechanical description an arbitrary initial state may be chosen presents great difficulties when the problem of atomic structure is considered. In fact, if we must reckon with an infinite number of continuously varying states of motion of the atoms, then we come into obvious contradiction with our experimental knowledge that the elements possess definite properties. One might believe perhaps that the properties of the elements do not inform us directly of the behaviour of individual atoms but, rather, that we are always concerned only with statistical regularities holding for the average conditions of a large number of atoms. In the mechanical theory of heat, which not only permits of our accounting for the fundamental laws of thermodynamics, but also gives us an understanding of many of the general properties of matter, we have a well-known example of the fruitfulness of statistical mechanical considerations in the atomic theory. The elements have, however, other properties which permit of more direct conclusions being drawn with respect to the states of motion of the atomic constituents. Above all, we must assume that the quality of the light which the elements in certain circumstances emit and which is characteristic of each element is essentially determined by what occurs in a single atom. Just as the wireless waves tell us about the nature of the electrical oscillations in the apparatus of the broadcasting station, so should we expect, on the basis of the electromagnetic theory of light, that the frequencies of the individual lines in the characteristic spectra of the elements should give us information as to the motions of the electrons within the atom. However, mechanics does not offer us a sufficient basis for interpreting this information; indeed, owing to the possibility of a continuous variation of the mechanical states of motion mentioned above, it is not possible even to understand the occurrence of sharp spectral lines.

The missing element in our description of nature, evidently required to account for the behaviour of the atoms, has been supplied, however, by Planck's discovery of the so-called quantum of action. This discovery had its origin in his investigation of black body radiation, which, because of its independence of the special properties of the substances, offered a decisive test of the range of validity of the mechanical theory of heat and of the electromagnetic theory of radiation. It was the very inability of these theories to account for the law of black body radiation which led Planck to recognize a general feature of the laws of nature that had hitherto remained unnoticed. This feature, to be sure, is not obvious in the description of ordinary physical phenomena, but it has, nevertheless, caused a complete revolution in our account of such effects which depend on individual atoms. Thus, in contrast with the demand of continuity which characterizes the customary description of nature, the indivisibility of the quantum of action requires an essential element of discontinuity in the description of atomic phenomena. The difficulty of combining the new knowledge with our ordinary scheme of physical ideas became especially apparent through the discussion of the nature of light, which was renewed by Einstein in connection with his explanation of the photo-electric effect, although the question, judging from all earlier experimental results, had found a perfectly satisfactory solution within the frame of the electromagnetic theory. The situation which we meet here is characterized by the fact that we are apparently forced to choose between two mutually contradictory conceptions of the propagation of light: one, the idea of light waves, the other, the corpuscular view of the theory of light quanta, each conception expressing fundamental aspects of our experience. As we shall see in the following, this apparent dilemma marks a peculiar limitation of our forms of perception which is bound up with the quantum of action. This limitation is brought to light by a closer analysis of the applicability of the basic physical concepts in describing atomic phenomena.

Indeed, only by a conscious resignation of our usual demands for visualization and causality was it possible to make Planck's discovery fruitful in explaining the properties of the elements on the basis of our knowledge of the building stones of atoms. Taking the indivisibility of the quantum of action as a starting-point, the author suggested that every change in the state of an atom should be regarded as an individual process, incapable of more detailed description, by which the atom goes over from one so-called stationary state into another. According to this view, the spectra of the elements do not give us immediate information about the motions of the atomic parts, but each spectral line is associated with a transition process between two stationary states, the product of the frequency and the quantum of action giving the energy change of the atom in the process. In this way, it proved possible to obtain a simple interpretation of the general spectroscopic laws which Balmer, Rydberg and Ritz had succeeded in deriving from the experimental data. This view of the origin of spectra was directly supported also by the well-known experiments of Franck and Hertz on collisions between atoms and free electrons. The amounts of energy which can be exchanged in such collisions were found to agree exactly with the energy differences, computed from the spectra, between the stationary state in which the atom was before the collision and one of the stationary states in which it can exist after the collision. On the whole, this point of view offers a consistent way of ordering the experimental data, but the consistency is admittedly only achieved by the renunciation of all attempts to obtain a detailed description of the individual transition processes. We are here so far removed from a causal description that an atom in a stationary state may in general even be said to possess a free choice between various possible transitions to other stationary states. From the very nature of the matter, we can only employ probability considerations to predict the occurrence of the individual processes, which fact, as Einstein has emphasized, exhibits a close similarity to the conditions holding for the spontaneous radioactive transformations.

A peculiar feature of this attack on the problem of atomic structure is the extensive use of whole numbers which also play an important rôle in the empirical spectroscopic laws. Thus, the classification of stationary states, besides depending upon the atomic number, also depends on the so-called quantum numbers, to the systematics of which Sommerfeld has contributed so much. On the whole, the views considered have permitted us to account, to a considerable extent, for the properties and relationships of the elements on the basis of our general conceptions of atomic structure. Considering the great departure from our customary physical ideas, one might wonder that such an account has been possible, since, after all, our entire knowledge of the building stones of the atoms rests upon these ideas. Indeed, any use of concepts like mass and electric charge is obviously equivalent to the invocation of mechanical and electrodynamical laws. A method of making such concepts useful in other fields than that in which the classical theories are valid has been found, however, in the demand of a direct concurrence of the quantum-mechanical description with the customary description in the border region where the quantum of action may be neglected. The endeavours to utilize in the quantum theory every classical concept in a reinterpretation which fulfils this demand without being at variance with the postulate of the indivisibility of the quantum of action, found their expression in the so-called correspondence principle. However, there were many difficulties to overcome before a complete description based on the correspondence principle was actually accomplished, and only in recent years has it been possible to formulate a coherent quantum mechanics which may be regarded as a natural generalization of the classical mechanics, and in which the continuous, causal description is replaced by a fundamentally statistical mode of description.

A decisive step towards the attainment of this goal was made by the young German physicist, Werner Heisenberg, who showed how the ordinary ideas of motion may be replaced in a consistent way by a formal application of the classical laws of motion, the quantum of action appearing only in certain rules of calculation holding for the symbols which replace the mechanical quantities. This ingenious attack upon the problem of the quantum theory makes, however, great demands on our power of abstraction, and the discovery of new artifices which, in spite of their formal character, more closely meet our demands for visualization has, therefore, been of profound significance in the development and clarification of the quantum mechanics. I am referring to the ideas of matter waves, introduced by Louis de Broglie, which have proved so fruitful in the hands of Schrödinger, especially in connection with the conception of stationary states, the quantum numbers of which are interpreted as the number of nodes of the standing waves symbolizing these states. De Broglie's starting-point was the similarity, which had already been so important in the development of classical mechanics, existing between the laws governing the propagation of light and those holding for the motion of material bodies. In fact, the wave mechanics forms a natural counterpart to the above-mentioned light quantum theory of Einstein. As in this theory, so also in the wave mechanics, we are not dealing with a self-contained conceptual scheme but, rather, as especially emphasized by Born, with an expedient to formulate the statistical laws which govern atomic phenomena. It is true that the confirmation of the idea of matter waves, provided by the experiments on the reflection of electrons by metal crystals, is, in its way, just as decisive as the experimental evidence for the wave conception of the propagation of light. However, we must bear in mind that the application of matter waves is limited to those phenomena, in the description of which it is essential that the quantum of action be taken into account and which, therefore, lie outside the domain where it is possible to carry out a causal description corresponding to our customary forms of perception and where we can ascribe to words like "the nature of matter" and "the nature of light" meanings in the ordinary sense.

With the help of quantum mechanics, we master an extensive range of experience. Especially are we able to account for a large number of details concerning the physical and chemical properties of the elements. Recently, it has been possible even to obtain an interpretation of the radioactive transformations, in which the empirical probability laws holding for these processes appear as an immediate consequence of the peculiar statistical mode of description that characterizes the quantum theory. This interpretation provides an excellent example of the fruitfulness as well as of the formal nature of the wave conceptions. On one hand, we have here a direct connection with the customary ideas of motion, since, owing to the great energy of the fragments expelled by the atomic nuclei, the paths of these particles may be directly observed. On the other hand, the ordinary mechanical conceptions completely fail to provide us with a description of the course of the disintegration process, since the field of force surrounding the atomic nucleus would, according to these ideas, prevent the particles from escaping from the nucleus. On the quantum mechanics, however, the state of affairs is quite different. Though the field of force is still a hindrance which, for the most part, holds the matter waves back, yet it permits a small fraction of them to leak through. The part of the waves which escapes in this way in a certain time gives us a measure of the probability that the disruption of the atomic nucleus takes place during this time. The difficulty of speaking of "the nature of matter" without the above-mentioned proviso could scarcely be more strikingly brought to light.

In the case of the idea of light quanta, there exists a similar relationship between our conceptual pictures and the calculation of the probability of occurrence of the observable light effects. In accordance with the classical electromagnetic conceptions, we cannot, however, ascribe any proper material nature to light, since observation of light phenomena always depends on a transfer of energy and momentum to material particles. The tangible content of the idea of light quanta is limited, rather, to the account which it enables us to make of the conservation of energy and momentum. It is, after all, one of the most peculiar features of quantum mechanics that, in spite of the limitation of the classical mechanical and electromagnetic conceptions, it is possible to maintain the conservation laws of energy and momentum. In certain respects, these laws form a perfect counterpart to the assumption, basic for the atomic theory, of the permanence of the material particles, which is strictly upheld in the quantum theory even though the conceptions of motion are renounced.

As with classical mechanics, so quantum mechanics, too, claims to give an exhaustive account of all phenomena which come within its scope. Indeed, the inevitability of using, for atomic phenomena, a mode of description which is fundamentally statistical arises from a closer investigation of the information which we are able to obtain by direct measurement of these phenomena and of the meaning which we may ascribe, in this connection, to the application of the fundamental physical concepts. On one hand, we must bear in mind that the meaning of these concepts is wholly tied up with customary physical ideas. Thus, any reference to space-time relationships presupposes the permanence of the elementary particles, just as the laws of the conservation of energy and momentum form the basis of any application of the concepts of energy and momentum. On the other hand, the postulate of the indivisibility of the quantum of action represents an element which is completely foreign to the classical conceptions; an element which, in the case of measurements, demands not only a finite interaction between the object and the measuring instrument but even a definite latitude in our account of this mutual action. Because of this state of affairs, any measurement which aims at an ordering of the elementary particles in time and space requires us to forego a strict account of the exchange of energy and momentum between the particles and the measuring rods and clocks used as a reference system. Similarly, any determination of the energy and the momentum of the particles demands that we renounce their exact co-ordination in time and space. In both cases, the invocation of classical ideas, necessitated by the very nature of measurement, is, beforehand, tantamount to a renunciation of a strictly causal description. Such considerations lead immediately to the reciprocal uncertainty relations set up by Heisenberg and applied by him as the basis of a thorough investigation of the logical consistency of quantum mechanics. The fundamental indeterminacy which we meet here may, as the writer has shown, be considered as a direct expression of the absolute limitation of the applicability of visualizable conceptions in the description of atomic phenomena, a limitation that appears in the apparent dilemma which presents itself in the question of the nature of light and of matter.

The resignation as regards visualization and causality, to which we are thus forced in our description of atomic phenomena, might well be regarded as a frustration of the hopes which formed the starting-point of the atomic conceptions. Nevertheless, from the present standpoint of the atomic theory, we must consider this very renunciation as an essential advance in our understanding. Indeed, there is no question of a failure of the general fundamental principles of science within the domain where we could justly expect them to apply. The discovery of the quantum of action shows us, in fact, not only the natural limitation of classical physics, but, by throwing a new light upon the old philosophical problem of the objective existence of phenomena independently of our observations, confronts us with a situation hitherto unknown in natural science. As we have seen, any observation necessitates an interference with the course of the phenomena, which is of such a nature that it deprives us of the foundation underlying the causal mode of description. The limit, which nature herself has thus imposed upon us, of the possibility of speaking about phenomena as existing objectively finds its expression, as far as we can judge, just in the formulation of quantum mechanics. However, this should not be regarded as a hindrance to further advance; we must only be prepared for the necessity of an ever extending abstraction from our customary demands for a directly visualizable description of nature. Above all, we may expect new surprises in the domain where the quantum theory meets with the theory of relativity and where unsolved difficulties still stand as a hindrance to a complete fusion of the extension of our knowledge and of the expedients to account for natural phenomena which these theories have given us.

Even though it be at the end of the lecture, yet I am glad to have the opportunity of emphasizing the great significance of Einstein's theory of relativity in the recent development of physics with respect to our emancipation from the demand for visualization. We have learned from the theory of relativity that the expediency of the sharp separation of space and time, required by our senses, depends merely upon the fact that the velocities commonly occurring are small compared with the velocity of light. Similarly, we may say that Planck's discovery has led us to recognize that the adequacy of our whole customary attitude, which is characterized by the demand for causality, depends solely upon the smallness of the quantum of action in comparison with the actions with which we are concerned in ordinary phenomena. While the theory of relativity reminds us of the subjective character of all physical phenomena, a character which depends essentially upon the state of motion of the observer, so does the linkage of the atomic phenomena and their observation, elucidated by the quantum theory, compel us to exercise a caution in the use of our means of expression similar to that necessary in psychological problems where we continually come upon the difficulty of demarcating the objective content. Hoping that I do not expose myself to the misunderstanding that it is my intention to introduce a mysticism which is incompatible with the spirit of natural science, I may perhaps in this connection remind you of the peculiar parallelism between the renewed discussion of the validity of the principle of causality and the discussion of a free will which has persisted from earliest times. Just as the freedom of the will is an experiential category of our psychic life, causality may be considered as a mode of perception by which we reduce our sense impressions to order. At the same time, however, we are concerned in both cases with idealizations whose natural limitations are open to investigation and which depend upon one another in the sense that the feeling of volition and the demand for causality are equally indispensable elements in the relation between subject and object which forms the core of the problem of knowledge.

Before I conclude, it would be natural, at such a joint meeting of natural scientists, to touch upon the question as to what light can be thrown upon the problems regarding living organisms by the latest development of our knowledge of atomic phenomena which I have here described. Even though it may not yet be possible to give an exhaustive answer to this question, we can perhaps already catch a glimpse of a certain connection between these problems and the ideas of the quantum theory. A first hint in this direction we find in the circumstance that the mutual action between the organisms and the external world, upon which the sense impressions depend, may, at any rate in certain circumstances, be so small that it approaches the quantum of action. As it has often been remarked, a few light quanta are sufficient to produce a visual impression. We see, therefore, that the needs of the organism for independence and sensibility are here satisfied to the utmost limit permitted by the laws of nature, and we must be prepared to come upon similar conditions also at other points of decisive significance for the formulation of biological problems. If, however, the physiological phenomena exhibit a refinement which is developed to the abovementioned limit, then, indeed, this means that we at the same time approach the limit for an unambiguous description of them with the help of our ordinary visualizable conceptions. This in no way contradicts the fact that the living organisms to a wide extent present problems to us which lie within the range of our visualizable forms of perception and have formed a fruitful field for the application of physical and chemical points of view. Neither do we see any immediate limit for the applicability of these view-points. Just as we do not need to distinguish, in principle, between the current in a water pipe and the flow of blood in the vessels, no more should we expect, beforehand, any profound fundamental difference between the propagation of sense impressions in the nerves and the conduction of electricity in a metal wire. It is true, for all such phenomena, that a detailed account carries us into the domain of atomic physics; indeed, so far as the conduction of electricity is concerned, we have just learned, in quite recent years, that only that limitation of our visualizable conceptions of motion, which is characteristic of the quantum theory, enables us to understand how the electrons can make their way between the metal atoms of the wire. However, in the case of these phenomena, such a refinement in the mode of description is not necessary to account for those effects which first call for our consideration. With regard to the more profound biological problems, however, in which we are concerned with the freedom and power of adaptation of the organism in its reaction to external stimuli, we must expect to find that the recognition of relationships of wider scope will require that the same conditions be taken into consideration which determine the limitation of the causal mode of description in the case of atomic phenomena. Besides, the fact that consciousness, as we know it, is inseparably connected with life ought to prepare us for finding that the very problem of the distinction between the living and the dead escapes comprehension in the ordinary sense of the word. That a physicist touches upon such questions may perhaps be excused on the ground that the new situation in physics has so forcibly reminded us of the old truth that we are both onlookers and actors in the great drama of existence.